Nucleon to Delta Weak Excitation Amplitudes in the Non-relativistic Quark Model

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We investigate the nucleon to Delta(1232) vector and axial vector amplitudes in the non-relativistic quark model of the Isgur-Karl variety. A particular interest is to investigate the SU(6) symmetry breaking, due to color hyperfine interaction. We compare the theoretical estimates to recent experimental investigation of the Adler amplitudes by neutrino scattering.

I. INTRODUCTION

Recent claim [1] by a group of experimentalists at Los Alamos that they may be observing neutrino flavor oscillations is a dramatic example of high topical interest in the low and medium energy neutrino physics. Such experiments are now possible, for example, at Brookhaven, Fermilab and Los Alamos, just to name a few laboratories among many facilities around the world. As a by-product of this experimental opportunity of having excellent medium energy neutrino beams ($E_\nu \sim 1-3\ GeV$) being readily available, a rebirth of exploring with neutrinos the properties of hadrons, particularly nucleons, both in their ground states and the resonance region, is expected. There is a long history of such investigations [2-18] in the Delta(1232) resonance region since the sixties, the most recent experiment [18] in the Delta region have been done at Brookhaven. Given the topical interest of the structure of hadrons from the QCD point of view, the exploration of nucleons and their excited states, *by both electromagnetic and weak probes*, merit special attention. In this exploration, the weak structure functions are difficult to determine, but they provide valuable information, often complementary to that obtained by the electromagnetic interaction.

Among all the excited states of the nucleon, the Delta(1232) is perhaps the best studied one [19,20], by strong, electromagnetic and even weak interactions, the last one being the focus of this paper. Along with the nucleon, the Delta is of fundamental interest to quantum chromodynamics (QCD) and its application to the problem of hadron structure. Until we learn how to use QCD rigorously to compute hadron properties, particularly in its low-energy (hence non-perturbative) domain, we must cope with models that are "QCD-inspired". One of the most successful examples of these imperfect constructs is the quark shell model (QM), in which the gluonic degrees of freedoms are replaced by effective potentials. The origin of this can be traced back to the pre-QCD sixties, when Gürsey, Radicati [20] and Sakita [21] described the nucleon and the Delta in the fundamental **56**-dimensional representation of the spin-flavor SU(6) symmetry group. In this limit, the nucleon and the Delta wave functions are $|N>=|\{56\}^2S_s>$ and $|\Delta>=|\{56\}^4S_s>$. They are degenerate in the symmetry limit. The degeneracy is lifted by the color hyperfine interaction [22]. Thus, in the Isgur-Karl QM, the wave functions of the nucleon and the Delta are [23,24] :

$$|N>=a_s|^2S_s>+a_{s'}|^2S_{s'}>+a_M|^2S_M>+a_D|^4D_M>+a_p|^2P_A>, \tag{1}$$

$$|\Delta>=b_s|^4S_s>+b_{s'}|^4S_{s'}>+b_D|^4D_s>+b_{D'}|^2D_M>, \tag{2}$$

where the a's and b's are determined by diagonalizing the QM Hamiltonian in the N=2 harmonic oscillator basis.

The purpose of this work is to investigate the process [2]

$$p+\nu_\mu \rightarrow \Delta^{++}+\mu^-, \tag{3}$$

in the framework of the non-relativistic QM, using wave functions (2). Despite a long history of theoretical (Refs.[2-14]) and experimental [15-18] investigations of the process, *no calculations of the amplitudes for the weak transitions are available in the literature in the context of these general wave-functions.* Our main objective in this paper is to remedy this. We shall focus in this paper on the all relevant helicity amplitudes. There are four transverse ones: $A^V_{1/2},\ A^V_{3/2}$ and $A^A_{1/2},\ A^A_{3/2}$, where V stands for vector and A represents axial vector. We shall also discuss the relevant longitudinal (scalar) weak amplitudes. From the electromagnetic processes [25]

$$N+\gamma \rightleftharpoons \Delta, \tag{4}$$

we know the vector helicity amplitudes A^V_α, both extracted from the experiments and as computed in various theoretical approaches, such as the quark model (QM), both non-relativistic [23,24] and relativized [26], bags [27], topological [13] and non-topological [28] solitons, and by the lattice gauge theoretic method [29]. Similarly thorough theoretical investigations on the axial vector amplitudes have been suggested in a recent investigation [14] in the framework of

QM. This is our objective here: to compute the vector and axial vector amplitudes in the framework of QM. We shall relate them to the recently extracted Adler [4] amplitudes from the Brookhaven neutrino experiment [18], which has studied the reaction

$$\nu_\mu + d \rightarrow \mu^- + \Delta^{++}(1232) + n_s, \tag{5}$$

n_s being spectator neutron. When necessary, we shall go back to the older experiments as well.

Our main goal is to investigate the success of the non-relativistic QM in reproducing the measured transition form factors for the process (3). In the case of (4), the phenomenologically extracted magnetic dipole ($M1$) and electric quadrupole ($E2$) amplitudes [25] at the real photon point are considerably *larger* than the values obtained in the QM. Their values away from the real photon point are not very well-known experimentally as yet. This is going to change with the advent of the electron facility called the CEBAF. The theoretical deficit of the transition magnetic amplitude, compared to the observed one, seems to be confirmed by the low-energy Compton scattering, where the resonant magnetic polarizability appears to be completely dominated [30] by the Delta contribution. One of our goals here is to see if the axial-vector analogue of the $M1$ amplitude in the process (3) can be reproduced in the non-relativistic QM. We shall also make an estimate of the axial-vector analogue of the E2 amplitudes, which would be zero in the SU(6) symmetry limit. Thus, its non-zero value would be a direct manifestation of the effect of the color magnetism, just as a non-zero value of the $E2$ amplitude is in the vector sector.

Remainder of this paper is organized as follows: We derive an effective Hamiltonian for the QM with vector and axial vector interactions in section II, wherein we also find a relation between transverse vector and axial vector amplitudes. The calculation of helicity amplitudes is done in section III. We find relations between the helicity amplitudes and Adler's form-factors in section IV, for the comparison of our model estimates with experiments. We collect some exact SU(6) relations in section V. Section VI contains the main results of our QM calculation and its comparison with experiments. In this section, we introduce an axial vector analogue of the $E2/M1$ ratio. The SU(6) breaking results in this ratio becoming non-zero. Neutrino experiments can give us an estimate for this ratio. Our conclusions are summarized in section VII.

II. THE QUARK TRANSITION HAMILTONIAN

The electroweak interaction at the quark level is incorporated in the Dirac equation in the usual fashion. We introduce the leptonic current l_μ for the weak process, which is the analogue of the electromagnetic vector potential A_μ:

$$\sum_{i=1}^{3} \left(p_{\mu,i}\gamma^\mu - m_i - \gamma^\mu \left(1-\gamma_5\right) l_{\mu,i}\right) u = 0, \tag{6}$$

so that the Hamiltonian is

$$H = \sum_{i=1}^{3} \left(\alpha \cdot \left(\mathbf{p}_i - \left(1-\gamma_5\right)\mathbf{l}_i\right) + \beta m_i + \left(1-\gamma_5\right) l_{0,i}\right). \tag{7}$$

By doing a free Foldy-Wouthuysen reduction [31], the quark Hamiltonian can be truncated to

$$H = \sum_{i=1}^{3} \left(m_i + \frac{p^2}{2m_i}\right) + H^V_{int} - H^A_{int}, \tag{8}$$

with

$$H^V_{int} = a^V \sum_{i=1}^{3} \left(l_{0i} - \frac{1}{2m_i}\left(\mathbf{p}_i \cdot \mathbf{l}_i + \mathbf{l}_i \cdot \mathbf{p}_i\right) - \frac{1}{2m_i}\sigma \cdot \left(\nabla \times \mathbf{l}_i\right) + O\left(m_i^{-2}\right)\right), \tag{9}$$

$$H^A_{int} = a^A \sum_{i=1}^{3} \left(-\sigma \cdot \mathbf{l}_i + \frac{1}{2m_i}\sigma \cdot \left(\mathbf{p}_i\, l_{0i} + l_{0i}\mathbf{p}_i\right) + O\left(m_i^{-2}\right)\right). \tag{10}$$

Here i is the quark index, a^V and a^A are factors, which, in general, can be different. Unless otherwise stated, we shall take these factors to be unity. We take quark masses m_i to be the same, m, for up and down quarks. We can see that

the H_{int}^V is the same as interaction Hamiltonian of the electromagnetic interaction except for the obvious difference in coupling constants. The lepton current l_μ can be defined, in analogy to the electromagnetic vector potential A_μ, as follows:

$$\mathbf{l} = \sqrt{\frac{4\pi\alpha_W}{2K_0}} \sum_{k,\lambda} \left(\epsilon_\lambda a_{k\lambda} e^{i\mathbf{k}\cdot\mathbf{r}} + \epsilon_\lambda^* a_{k\lambda}^+ e^{-i\mathbf{k}\cdot\mathbf{r}} \right), \tag{11}$$

$$l_0 = \sqrt{\frac{4\pi\alpha_W}{2K_0}} \sum_{k,\lambda} \left(a_{k\lambda} e^{i\mathbf{k}\cdot\mathbf{r}} + a_{k\lambda}^+ e^{-i\mathbf{k}\cdot\mathbf{r}} \right), \tag{12}$$

where we introduce a "weak" fine structure constant, α_W, for emitting or absorbing a weak quantum, W, with four-momentum q_μ $(k_0; 0, 0, k)$. Here K_0 is the energy transfer for $q_\mu q^\mu = 0$, given by

$$K_0 = \frac{M_\Delta^2 - M^2}{2M_\Delta} \approx 255.8 MeV. \tag{13}$$

This is the analogue of the real photon point. We use K_0 in the normalization factor under the radical sign in Eq.(11,12), a practice adopted by many [26,32]. This factor cancels out in the form-factor expression, having no influence on the form-factors we shall calculate. For the transverse amplitude, we use the boson polarization vector to be $\epsilon = -\frac{1}{\sqrt{2}}(1, i, 0)$. The vector transverse Hamiltonian can be separated into two pieces:

$$H_{int,\, trans.}^V = H_I^V + H_{II}^V, \tag{14}$$

with

$$H_I^V = \sqrt{\frac{4\pi\alpha_W}{2K_0}} \frac{k}{m\sqrt{2}} \left(3\frac{(\tau_3)^{(3)}}{2} \right) \left(s_x^{(3)} + i s_y^{(3)} \right) \exp\left(-i\sqrt{\frac{2}{3}} k\lambda_z \right), \tag{15}$$

$$H_{II}^V = \sqrt{\frac{4\pi\alpha_W}{2K_0}} \frac{1}{m\sqrt{2}} \left(3\frac{(\tau_3)^{(3)}}{2} \right) \sqrt{\frac{2}{3}} \left(p_{\lambda_x}^{(3)} + i p_{\lambda_y}^{(3)} \right) \exp\left(-i\sqrt{\frac{2}{3}} k\lambda_z \right), \tag{16}$$

taking $\frac{\sigma}{2} = \mathbf{s}$.

The correspondence between the usual photon case and the virtual W-boson exchange discussed above, needed for the process (3), can be seen by the appropriate identification of the photon variables to the W-boson variables. For transverse axial transition, l_0 does not contribute, and the other term is

$$H_{trans.}^A = \sqrt{\frac{4\pi\alpha}{2K_0}} \sqrt{2} \left(3\frac{(\tau_3)^{(3)}}{2} \right) \left(s_x^{(3)} + i s_y^{(3)} \right) \exp\left(-i\sqrt{\frac{2}{3}} k\lambda_z \right). \tag{17}$$

Thus we have an important relation

$$H_{trans.}^A = \frac{2m}{k} \frac{e^A}{e^V} H_I^V. \tag{18}$$

where e^A/e^V, denote the possible coupling difference between vector and axial vector parts. If we compare directly with photon transitions, e^V is simply e.

III. CALCULATION OF THE HELICITY AMPLITUDES

The calculation of the longitudinal and scalar amplitudes requires a discussion of the possible lack of current conservation for chiral vector and axial vector currents in the QM space chosen.. Though discussions on this issue have been made from time to time in the literature [8,32], this issue is not settled in our opinion. In this paper, we simply avoid tackling this issue. We begin here with a discussion of transverse helicity amplitudes. The leading orders of these amplitudes do not suffer from the uncertainties of the lack of current conservation.

The calculation of the transverse helicity amplitudes for the vector current is standard [23,24,32]. The wave functions of Δ and N, taking the effects of the color-hyperfine interaction, can be expressed in terms of SU(6) basis functions as

$$|\Delta >= 0.97|^4S_s > +0.20|^4S'_s > -0.097|^4D_s > +0.065|^2D_M >, \tag{19}$$

$$|N >= 0.95|^2S_s > -0.24|^2S'_s > -0.20|^2S_M > -0.042|^4D_M >, \tag{20}$$

ignoring the tiny contribution from $|^2P_A >$ in (1). These two wave functions correspond [24] to energy levels $E_\Delta = 1230MeV$, and $E_N = 940MeV$. The transverse helicity amplitudes for the electromagnetic current are defined in the photon-nucleon CM frame [24]:

$$A^V_\lambda =< \Delta, M_J = \lambda|H^V_{int}|\gamma N, \lambda = \lambda_\gamma - \lambda_N, \mathbf{k} >, \tag{21}$$

where the helicity λ is 3/2 or 1/2. These equations are straightforwardly generalizable to the case of our interest, introducing virtual W boson mediating the charged weak current that produces the reaction (3). From these quantities, we can define the familiar [23,24] transverse electromagnetic multipoles:

$$M1 = -\frac{1}{2\sqrt{3}}\left(3A^V_{3/2} + \sqrt{3}A^V_{1/2}\right) \tag{22}$$

$$E2 = \frac{1}{2\sqrt{3}}\left(A^V_{3/2} - \sqrt{3}A^V_{1/2}\right) \tag{23}$$

We can also define the longitudinal and scalar amplitudes L^V and S^V, for which the virtual boson or photon helicity is zero. Clearly, these two amplitudes are related [32]:

$$k_0S^V = kL^V, \tag{24}$$

a relation which can [32] be violated due to truncation of the model space in the QM. The relation (24) is a consequence of the conserved vector current (CVC), or equivalently, the gauge invariance of the vector current.

The calculation of the transverse, longitudinal (and scalar) amplitudes for the vector current have been done by many authors [23,24,32]. Thus we do not discuss them here. We shall give here a brief discussion for the axial vector amplitudes. The longitudinal axial transition operator is given by

$$H^{AL}_{int} = a\sum_{i=1}^{3} -\sigma_{iz}l_z, \tag{25}$$

where we are considering the axial vector term only, Eq.(10). Following (15), we have

$$H^{AL}_{int} = \sqrt{\frac{4\pi\alpha_W}{2K_0}}\left(-3\sigma^{(3)}_z\right)\exp\left(-i\sqrt{\frac{2}{3}}k\lambda_z\right). \tag{26}$$

From Eq.(10), the scalar term is

$$H^{AS}_{int} = a\sum_{i=1}^{3}\frac{1}{2m_i}\left\{\sigma_i\cdot\mathbf{p}_i,\, l_{0i}\right\}. \tag{27}$$

Following (16), we have

$$\begin{gathered} H^{AS}_{int} = \sqrt{\tfrac{4\pi\alpha_W}{2K_0}} \\ \times\left(\tfrac{1}{m\sqrt{3}}\left(p_{\lambda_+}\sigma^{(3)}_- + p_{\lambda_-}\sigma^{(3)}_+\right) - \tfrac{k}{m}\left[\tfrac{n_\Delta - n_N}{\overline{k}^2} - \tfrac{1}{6}\right]\sqrt{2}\sigma^{(3)}_z\right)\exp\left(-i\sqrt{\tfrac{2}{3}}k\lambda_z\right), \end{gathered} \tag{28}$$

where $\overline{k} = k/\alpha_{HO}$, α_{HO} being the harmonic oscillator parameter of the IK model, which has a value [23] of 320MeV from the fitting of the nucleon spectra. Here n_Δ and n_N are the principal quantum numbers of Δ and nucleon. The longitudinal and scalar amplitudes are defined respectively as

$$L^A = <\Delta; J_z = \frac{1}{2}|H_{int}^{AL}|N; J_z = \frac{1}{2}>, \tag{29}$$

$$S^A = - <\Delta; J_z = \frac{1}{2}|H_{int}^{AS}|N; J_z = \frac{1}{2}> . \tag{30}$$

These amplitudes are shown in Table I.

We shall now use the computed values of $A_{3/2}^V$ and $\sqrt{3}A_{1/2}^V$, obtained with $H_I^{(1)}$ of Eq.(14), and apply the relation (17) to compute the $A_{3/2}^A$ and $\sqrt{3}A_{1/2}^A$. The results are shown in Table II. In analogy to $M1$ and $E2$, discussed earlier (Eqs.(22,23)), we can also compute the amplitudes $(M1)^A$ and $(E2)^A$, in the form:

$$(M1)^A = A(\overline{k})f_M^A(\overline{k}),\ (E2)^A = A(\overline{k})f_E^A(\overline{k}), \tag{31}$$

Here

$$A(\overline{k}) = \sqrt{\frac{4\pi\alpha_W}{2K_0}}e^{-\overline{k}^2/6}, \tag{32}$$

and $f_M^A(\overline{k})$, $f_E^A(\overline{k})$ are model-dependent, to be explicitly given later.

QM calculations of $A_{1/2}^V$, $A_{3/2}^V$, L^V and S^V are standard in the literature. For completeness, we have collected these results in our notation in Table III. Bourdeau and Mukhopadhyay [32] have discussed the violation of (24) in the IK and other quark models. Numerical results of the form-factors from IK model are shown in section VI, subsection D. Ratios of helicity amplitudes are shown in Figs. 1 and 2. Comparison with experiments is done through the Adler form-factors discussed below.

IV. THE ADLER-RARITA-SCHWINGER FORMALISM

The standard method in the theoretical treatment of weak interaction process in (3) follows the Rarita-Schwinger [33,34] formalism and the $N \rightarrow \Delta$ transition form factors are introduced following a notation due to Adler [4] and Llewellyn Smith [2]. Thus, the invariant matrix element is

$$\mathbf{M} = <\mu^-\Delta^{++}|H_{int}|\nu p> = \frac{G_F\cos\theta}{\sqrt{2}}j_\alpha <\Delta^{++}|V^\alpha - A^\alpha|p> . \tag{33}$$

where G$_F$ the Fermi constant, and θ is the Cabibbo angle,

$$j^\alpha = \overline{u}_\mu\gamma^\alpha(1-\gamma^5)u_\nu \tag{34}$$

is the weak lepton current. We can decompose the invariant matrix element as [2,4,10]

$$\begin{aligned}\frac{\mathbf{M}}{\sqrt{3}} = \frac{G}{\sqrt{2}}\overline{\psi}_\alpha\{&\left(\frac{C_3^V}{M}\gamma_\lambda + \frac{C_4^V}{M^2}(P_\Delta)_\lambda + \frac{C_5^V}{M^2}(P_p)_\lambda\right)\gamma_5F^{\lambda\alpha} + C_6^V j^\alpha\gamma_5\\ &+\left(\frac{C_3^A}{M}\gamma_\lambda + \frac{C_4^A}{M^2}(P_\Delta)_\lambda\right)F^{\lambda\alpha} + C_5^A j^\alpha + \frac{C_6^A}{M^2}(q)^\alpha q^\lambda j_\lambda\}u,\end{aligned} \tag{35}$$

where

$$F^{\lambda\alpha} = q^\lambda j^\alpha - q^\alpha j^\lambda, \tag{36}$$

P_Δ is the Delta four-momentum, ψ_μ is the Delta vector spinor and u, the proton spinor. The $C_i^{V,A}$ are the so-called Adler form-factors [4]. These form factors can be related to the helicity amplitudes through calculation of $\mathbf{M}$ projected in different polarizations. The factor $\sqrt{3}$ in Eq.(35) above comes from the isospin relations, such as that between the weak and electromagnetic matrix elements:

$$<\Delta^{++}|V^\alpha|p> = \sqrt{3} <\Delta^+|V_{EM}^\alpha|p> . \tag{37}$$

After separating j_α, we get

$$\frac{1}{\sqrt{3}} < \Delta^{++}| - A^\alpha|p >= \overline{\psi}_\alpha \left(\frac{C_3^A}{M}\gamma_\lambda + \frac{C_4^A}{M^2}(P_\Delta)_\lambda \right) q^\lambda u$$

$$- \overline{\psi}_\lambda \left(\frac{C_3^A}{M}\gamma_\alpha + \frac{C_4^A}{M^2}(P_\Delta)_\alpha \right) q^\lambda u + \overline{\psi}_\alpha C_5^A u + \overline{\psi}_\lambda \frac{C_6^A}{M^2} q^\lambda q_\alpha u. \tag{38}$$

Recent experiments on the process (3) at the Brookhaven National Laboratory [18] have been analyzed in terms of the Adler form factors. Thus, we must find relations between the helicity amplitudes, discussed above, and the Adler amplitudes.

In the Δ rest frame, which is also the WN(or γN) CM frame,

$$(P_\Delta)_0 = M_\Delta = E_N + k_0 = \sqrt{M^2 + \mathbf{k}^2} + k_{0,} \tag{39}$$

$\mathbf{p}_\Delta = 0$ and $q^\lambda q_\lambda = k_0^2 - \mathbf{k}^2$. We also define the Delta vector spinors as

$$\psi_{3/2}^\Delta = \epsilon_+ \chi_+, \tag{40}$$

$$\psi_{1/2}^\Delta = \frac{1}{\sqrt{3}}\epsilon_+\chi_- + \sqrt{\frac{2}{3}}\epsilon_0\chi_+. \tag{41}$$

Here the polarization vectors are

$$\epsilon_+ = -\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix},\ \epsilon_- = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix},\ \epsilon_0 = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{42}$$

Thus, the vector spinor

$$\psi_{3/2}^\Delta = \epsilon_+\chi_+ = \left\{ 0; -\frac{1}{\sqrt{2}}\begin{pmatrix} \chi_+ \\ 0 \end{pmatrix}, -\frac{i}{\sqrt{2}}\begin{pmatrix} \chi_+ \\ 0 \end{pmatrix}, 0 \right\}. \tag{43}$$

and so on.

The nucleon spinors are

$$u_\pm^N = \begin{pmatrix} \chi^\pm \\ \frac{\sigma\cdot\mathbf{p}_N}{E_N+M}\chi^\pm \end{pmatrix} = \begin{pmatrix} \chi^\pm \\ -\frac{\sigma\cdot\mathbf{k}}{E_N+M}\chi^\pm \end{pmatrix}. \tag{44}$$

Thus we can derive the following relations:

$$\begin{gathered} -A_{3/2}^V = \sqrt{\tfrac{4\pi\alpha_W}{2K_0}}\tfrac{k}{E_N+M} \\ \times \left[\tfrac{E_N+M+k_0}{M}C_3^V + \tfrac{k_0 M_\Delta}{M^2}C_4^V + \tfrac{k_0E_N+\mathbf{k}^2}{M^2}C_5^V + C_6^V \right], \end{gathered} \tag{45}$$

$$\begin{gathered} \sqrt{3}A_{1/2}^V = \sqrt{\tfrac{4\pi\alpha_W}{2K_0}}\tfrac{k}{E_N+M} \\ \times \left[\tfrac{-E_N-M+k_0}{M}C_3^V + \tfrac{k_0 M_\Delta}{M^2}C_4^V + \tfrac{k_0E_N+\mathbf{k}^2}{M^2}C_5^V + C_6^V \right], \end{gathered} \tag{46}$$

$$\sqrt{\frac{3}{2}}L^V = \sqrt{\frac{4\pi\alpha_W}{2K_0}}\frac{k}{E_N+M}\left[\frac{k_0}{M}C_3^V + \frac{k_0M_\Delta}{M^2}C_4^V + \frac{k_0E_N}{M^2}C_5^V + C_6^V \right], \tag{47}$$

$$\sqrt{\frac{3}{2}}S^V = \sqrt{\frac{4\pi\alpha_W}{2K_0}}\frac{k}{E_N+M}\left[\frac{k}{M}C_3^V + \frac{kM_\Delta}{M^2}C_4^V + \frac{kE_N}{M^2}C_5^V \right], \tag{48}$$

$$-A^A_{3/2}=\sqrt{\frac{4\pi\alpha_W}{2K_0}}\left[\left(k_0+\frac{\mathbf{k}^2}{E_N+M}\right)\frac{1}{M}C^A_3+\frac{k_0M_\Delta}{M^2}C^A_4+C^A_5\right], \tag{49}$$

$$-\sqrt{3}A^A_{1/2}=\sqrt{\frac{4\pi\alpha_W}{2K_0}}\left[\left(k_0-\frac{\mathbf{k}^2}{E_N+M}\right)\frac{1}{M}C^A_3+\frac{k_0M_\Delta}{M^2}C^A_4+C^A_5\right], \tag{50}$$

$$-\sqrt{\frac{3}{2}}L^A=\sqrt{\frac{4\pi\alpha_W}{2K_0}}\left[\frac{k_0}{M}C^A_3+\frac{k_0M_\Delta}{M^2}C^A_4+C^A_5-\frac{\mathbf{k}^2}{M^2}C^A_6\right], \tag{51}$$

$$-\sqrt{\frac{3}{2}}S^A=\sqrt{\frac{4\pi\alpha_W}{2K_0}}\left[\frac{k}{M}C^A_3+\frac{kM_\Delta}{M^2}C^A_4-\frac{k_0k}{M^2}C^A_6\right]. \tag{52}$$

From the relations for the vector amplitudes, we get

$$A^V_{3/2}+\sqrt{3}A^V_{1/2}=-\sqrt{\frac{4\pi\alpha_W}{2K_0}}\frac{2k}{M}C^V_3, \tag{53}$$

by far the *most important* vector contribution in the $N\to\Delta$ transition. We also get

$$\frac{1}{2}\left(-A^V_{3/2}+\sqrt{3}A^V_{1/2}\right)-\sqrt{\frac{3}{2}}L^V=\sqrt{\frac{4\pi\alpha_W}{2K_0}}\frac{k^3}{M^2(E_N+M)}C^V_5, \tag{54}$$

$$\sqrt{\frac{3}{2}}\left(kL^V-k_0S^V\right)=\sqrt{\frac{4\pi\alpha_W}{2K_0}}\left(E_N-M\right)C^V_6. \tag{55}$$

By CVC or gauge invariance, the left-hand side is zero identically. Thus we get a CVC relation, $C^V_6=0$, a relation *which will be mildly violated by the IK quark model calculation*, due to its inability to make the left-hand side of (55) vanish identically [32].

Rearranging the axial-vector transition helicity amplitudes, we have

$$\sqrt{\frac{4\pi\alpha_W}{2K_0}}C^A_3=\frac{M}{2\left(E_N-M\right)}\left(\sqrt{3}A^A_{1/2}-A^A_{3/2}\right). \tag{56}$$

We can define relations analogous to (22) and (23):

$$(M1)^A=-\frac{1}{2\sqrt{3}}\left(3A^A_{3/2}+\sqrt{3}A^A_{1/2}\right), \tag{57}$$

$$(E2)^A=\frac{1}{2\sqrt{3}}\left(A^A_{3/2}-\sqrt{3}A^A_{1/2}\right). \tag{58}$$

We also have the relations

$$\sqrt{\frac{4\pi\alpha_W}{2K_0}}C^A_3=-\sqrt{3}\frac{M}{2\left(E_N-M\right)}\left(E2\right)^A, \tag{59}$$

$$\sqrt{\frac{4\pi\alpha_W}{2K_0}}\left(C^A_5+\frac{M_\Delta k_0}{M}C^A_4\right)=-\frac{1}{2}\left(\sqrt{3}A^A_{1/2}+A^A_{3/2}\right)-\frac{k_0}{M}\sqrt{\frac{4\pi\alpha_W}{2K_0}}C^A_3, \tag{60}$$

$$=\frac{\sqrt{3}}{2}\left(\frac{k_0+E_N-M}{E_N-M}\left(E2\right)^A+\left(M1\right)^A\right). \tag{61}$$

The amplitudes S^A and L^A are connected by the identity

$$kL^A - k_0 S^A = -\sqrt{\frac{2}{3}} k \sqrt{\frac{4\pi\alpha_W}{2K_0}} \left[C_5^A + \frac{k_0^2 - \mathbf{k}^2}{M^2} C_6^A \right]. \tag{62}$$

We get the conservation of the axial current (CAC) in the chiral limit ($m_\pi \to 0$) in which this identity reduces to

$$kL^A - k_0 S^A = 0. \tag{62'}$$

We must examine, if the chiral symmetry breaking and the truncation of the model space in the QM results in the violation of the last relation. The answer we find below is in the affirmative.

Using (49-52) we can derive the following relations:

$$C_6^A = \frac{M^2}{\mathbf{k}^2} \sqrt{\frac{2K_0}{4\pi\alpha_W}} \left[-\frac{1}{2} \left(\sqrt{3} A_{3/2}^A + A_{1/2}^A \right) + \sqrt{\frac{3}{2}} L^A \right], \tag{63}$$

$$C_5^A = -\sqrt{\frac{3}{2}} \sqrt{\frac{2K_0}{4\pi\alpha_W}} \left(L^A - \frac{k_0}{k} S^A \right) - \frac{k_0^2 - \mathbf{k}^2}{M^2} C_6^A, \tag{64}$$

$$C_4^A = \frac{M^2}{kM_\Delta} \left[-\sqrt{\frac{3}{2}} \sqrt{\frac{2K_0}{4\pi\alpha_W}} S^A - \frac{k}{M} C_3^A + \frac{k_0 k}{M^2} C_6^A \right]. \tag{65}$$

Thus, Eqs. (56,63-65) complete the expressions for the four Adler axial nuclear to Delta form factors, in terms of the calculated helicity amplitudes.

The partial conservation of axial current (PCAC) relation can be expressed in another way, as discussed earlier by Schreiner and von Hippel [10]. From the pion pole dominance of the divergence of the axial current, taken between the nucleon and the Delta states, we get the induced pseudoscalar term given by the pion pole, exactly parallel to the weak current between nucleon states, wherein the pion pole term also yields the induced pseudoscalar term [10,14]. Thus,

$$\frac{C_6^A}{M^2} = \frac{g_\Delta f_\pi}{2\sqrt{3} M \left(m_\pi^2 - q^2 \right)}, \tag{66}$$

at the pion pole. Here g_Δ is the $\Delta^{++} \to p\pi^+$ coupling constant, recently redetermined by Davidson *et al.* [25]:

$$g_\Delta = 28.6 \pm 0.3\ . \tag{67}$$

Actually, the determination of g_Δ depends on which method we use in the analysis, thereby yielding a much bigger theoretical error than that is given in (67). In Eq.(66), f_π is the pion decay constant

$$f_\pi \approx 0.97 m_\pi. \tag{68}$$

Taking the limit of the divergence of the axial current as $m_\pi^2 \to 0$ and $q^2 \to 0$, we get the *off-diagonal* Goldberger-Treiman relation

$$C_5^A(0) = \frac{g_\Delta f_\pi}{2\sqrt{3} M}. \tag{69}$$

This off-diagonal cousin of the well-known *diagonal* Goldberger-Treiman relation has not been given much attention in the literature, except in the recent work by Hemmert *et al.* [14]. Using (69), the estimate of $C_5^A(0)$ is

$$C_5^A(0) \approx 1.2. \tag{70}$$

The relations (62') and (69) should be *simultaneously satisfied* if PCAC is to be valid.

We shall further discuss the subject of PCAC and its validity in the QM in subsection VI.B. Form-factors calculated here are shown in Figs. 3-10.

V. THE EXACT SU(6) SYMMETRY RELATIONS

Let us first start with the vector amplitudes. In the SU(6) symmetry limit [35],

$$M1 \neq 0,\ E2 = 0. \tag{71}$$

The Fermi-Watson theorem implies these multipoles to be purely imaginary on top of the Delta resonance. Thus,

$$Im(E2)/Im(M1) \equiv EMR = 0. \tag{72}$$

In the QM, all amplitudes are purely real, hence E2/M1=0 in the SU(6) limit, giving

$$A^V_{3/2}(SU(6)) = \sqrt{3}A^V_{1/2}(SU(6)). \tag{73}$$

Using this relation in Eq.(53), we get, in the SU(6) symmetry limit, the largest vector form factor, C^V_3, given by (Table II)

$$C^V_3(SU(6)) = -\left(\sqrt{\frac{4\pi\alpha_W}{2K_0}}\frac{k}{M}\right)^{-1} A^V_{3/2}(SU(6)) = \frac{M}{\sqrt{3}m}e^{-\overline{k}^2/6}. \tag{74}$$

Also, the left-hand side of (54) vanishes in the SU(6) limit, giving

$$C^V_5(SU(6)) = 0. \tag{75}$$

Likewise, we can get

$$C^V_4(SU(6)) = -\frac{M}{M_\Delta}C^V_3(SU(6)). \tag{76}$$

Finally, the longitudinal vector response vanishes in the SU(6) limit:

$$L^V(SU(6)) = S^V(SU(6)) = 0. \tag{77}$$

Thus, the only non-vanishing multipole vector amplitude in the SU(6) limit is the magnetic dipole amplitude, given by

$$M1(SU(6)) = -2A^V_{1/2}(SU(6)) = \frac{2k}{3m}A(\overline{k}). \tag{78}$$

We can similarly discuss the axial vector amplitude in the SU(6) limit. The only non-vanishing amplitude is

$$(M1)^A(SU(6)) = \frac{4}{3}A(\overline{k}), \tag{79}$$

and

$$(E2)^A(SU(6)) = 0. \tag{80}$$

The axial vector transverse helicity amplitudes are:

$$\frac{1}{\sqrt{3}}A^A_{3/2}(SU(6)) = A^A_{1/2}(SU(6)) = -\frac{2}{3}A(\overline{k}).$$

The scalar and longitudinal axial vector amplitudes are:

$$S^A(SU(6)) = \frac{\sqrt{2}}{9}\frac{k}{m}A(\overline{k}), \tag{81}$$

$$L^A(SU(6)) = -\frac{2\sqrt{2}}{3}A(\overline{k}). \tag{82}$$

The Adler form factors become

$$C_3^A(SU(6)) = C_6^A(SU(6)) = 0, \tag{83}$$

$$C_5^A(SU(6)) = \left(\frac{2}{\sqrt{3}} + \frac{1}{3\sqrt{3}}\frac{k_0}{m}\right) e^{-\overline{k}^2/6}, \tag{84}$$

$$C_4^A(SU(6)) = -\frac{1}{3\sqrt{3}}\frac{M^2}{M_\Delta m} e^{-\overline{k}^2/6}. \tag{85}$$

In Figs.1 and 2, we plot the ratios

$$r^b = \frac{A^b_{3/2}}{\sqrt{3}A^b_{1/2}} \tag{86}$$

as functions of $Q^2 = -q^2$, b=V, A, for the IK wave functions of the nucleon and the Delta, and the wave-functions [32] inspired by a model of Vento *et al.* (VBJ) [36]. In the SU(6) limit, this ratio should be unity for both V and A currents. The deviation from unity is due to the SU(6) breaking interactions. In the IK case, that is from the color hyperfine interaction.

The SU(6) relations described above are only approximate, since the SU(6) symmetry is broken by the color hyperfine interaction. Thus, the IK model wave functions and the actual experiments would violate these relations. The degree of these violations is an interesting question and lots of theoretical [25,36,37] and experimental [38] attention are being given to it, since the finding of Davidson *et al.* [25] that the EMR is not zero from the existing old electromagnetic data. We hope our study here of the axial vector amplitudes would trigger similar interest in the *weak sector*.

We note that the SU(6) symmetry limit to the nucleon and the Delta wave function provides consistency with the requirements of CVC. Thus, the CVC requirement of current conservation is trivially satisfied. The vanishing of C_6^V, required by the CVC limit, is again trivially true.

The CAC relation (62') is *not* satisfied in the SU(6) limit. However, we get good agreement with the off-diagonal Goldberger-Treiman relation, Eq.(69). Thus, going to the "real photon" point $k = k_0$, we can evaluate the $C_5^A(Q^2 = 0)$ by using (84). We get

$$C_5^A(Q^2 = 0)(SU(6)) = 1.17, \tag{87}$$

compared with the off-diagonal Goldberger-Treiman value of 1.2, a good agreement. We can also see that the identity (62) is satisfied in the SU(6) limit of QM, as it must. However, both sides are quite large at the real photon point:

$$\frac{\sqrt{2}}{3}K_0\sqrt{\frac{4\pi\alpha_W}{2K_0}}e^{-\overline{K}_0^2/6}\left(\frac{K_0}{3m} + 2\right), \tag{88}$$

instead of the PCAC expectation of both-sides of (62) vanishing at the chiral limit, Eq.(62').

VI. MAIN RESULTS OF THE QUARK MODEL AND COMPARISON WITH EXPERIMENTAL RESULTS

A. Comparison with other models

An extensive review of the old QM calculations by many authors has been done by Schreiner and von Hippel [10], who have provided a detailed comparison between theory and experiment, as available till 1973. The readers are referred to their paper for a discussion. In Table IV, we summarize the predictions of various form factors, vector and axial vector, in different models, and compare them with the new QM results reported here, as well as the recent estimates of Hemmert *et al.* [14].

B. CVC and PCAC

We have seen earlier that the quantity $\left(kL^V - k_0S^V\right)$, the left-hand side of Eq. (55), vanishes in the SU(6) limit, thereby satisfying the CVC. Other quark model wave functions for the nucleon and the Delta, such as those of

the Isgur-Karl model and the VBJ model [36], discussed below, do not satisfy this CVC constraint [32]. The CVC violations in the IK and VBJ models are demonstrated in Figs. 5 and 6 where the C_5^V and the C_6^V are plotted as functions of $-q^2$ for W=1230MeV and W=1234MeV.

As noted earlier, the PCAC pion pole estimate of $C_6^A(q^2)$, Eq.(66), is not fulfilled in the QM. The IK quark model, for example, produces a value of $C_6^A(q^2)$ *much smaller* than what Eq.(66) suggests. This failure is readily understandable, because the inclusion of the pion-pole term in a QM that involves only quark degrees of freedom *and no mesons*, is not legitimate. However, what is surprising is that the *off-diagonal Goldberger-Treiman relation* (Eq.(69)) *is well-satisfied in the SU(6) limit and by the IK quark model wave functions.* Thus, in the SU(6) limit, we have Eq.(87); in the IK model, we get

$$C_5^A(0)(IK) = 1.16. \tag{89}$$

This, however, occurs due to an accident. In the chiral limit

$$\left(L^A - \frac{k_0}{k} S^A\right)_{chiral} \rightarrow 0, \tag{90}$$

in Eq.(64), and C_6^A would pick up a PCAC contribution, not present in our QM calculations. The latter does not satisfy the chiral axial current conservation and does not pick up the PCAC contribution in C_6^A. However, these two theoretical inaccuracies somehow add up to the PCAC estimate in our SU(6) and IK quark model wave functions. Further theoretical work is needed to illuminate the nature of this happy accident.

While the degree of violation of CVC and PCAC should be as small as possible in the quark model of excellent quality, this does not mean we prefer the SU(6) limit to the IK model. That is because the color hyperfine interaction and other dynamical considerations make the SU(6) limit inaccurate for baryon spectroscopy. We must, therefore, search for a model beyond that limit. Between the IK model and the VBJ models, the former is clearly superior, as the violation of CVC is less in the former than in the latter, and the IK model account for the baryon spectroscopy, while the VBJ model is not so ambitious.

C. SU(6) breaking

We can go back to Figs. 1 and 2 to demonstrate the SU(6) breakings in the IK [23,24] and VBJ [36] models. These effects are significant, though difficult to measure experimentally. In Figs. 3 and 9, the SU(6) limits coincide with the theoretical predictions of CVC and off-diagonal Goldberger-Treiman relation. Here, more realistic quark models produce mild violation of these important constraints. Such violation can arise from the model truncation effects. Some authors have tried to remedy these with the introduction of the form factors at the quark level [8,11]. We shall not do that, as we believe this is not a satisfactory remedy in the spirit of QCD.

D. Comparison with experimental results

The experiments on neutrino scattering in the Delta resonance region are very difficult, nevertheless they have been done in a number of labs, ANL, CERN, Fermilab and BNL [15-18]. These experiments deal with single pion production in the charged current reaction in hydrogen and deuterium. In view of the extensive literature on the older experiments, we shall focus here on the most recent one.

Latest experimental results [18] come from the Brookhaven National Laboratory(BNL). The analysis by Kitagaki *et al.* [18]. makes some strong assumptions, including the neglect of the background contributions and use of the polynomial forms for the transition form factors, prescribed by Adler [4]. The dipole form they use is

$$F_V(Q^2) = \lambda(Q^2)/(1 + Q^2/M_V^2)^2 \tag{91}$$

where $\lambda(Q^2)$ is a function introduced by Olsson *et al.* [39]:

$$\lambda^2(Q^2) = 1 - (0.053 + 0.017Q)\sin\left(\frac{4.00Q}{1 + 0.22Q}\right). \tag{92}$$

Kitagaki *et al.* [18] find, from a fit to their new data,

$$M_V = 0.89^{+0.04}_{-0.07}\, GeV. \tag{93}$$

We demonstrate the results from Kitagaki *et al.* by displaying $C_3^V(Q^2)$, using Adler parameters [4], in Fig. 3 as a function of Q^2. We also plot the earlier experimental fit by Dufner and Tsai [40], who got a good fit to the data on the electroproduction of pions in the Delta region, by using the form

$$|C_3^V|^2 = (2.05)^2 \left(1 + 9(Q^2)^{1/2}\right) \exp\left(-6.3(Q^2)^{1/3}\right). \tag{94}$$

We shall also use the simpler dipole form [10], which fits the above data just as well:

$$C_3^V = 2.05 \left(1 + \frac{Q^2}{0.54}\right)^{-2}. \tag{95}$$

In the formulae above, Q^2 is in GeV^2. We notice that the IK model [23,24] does a reasonable job of describing the experimental data on C_3^V, except at $Q^2 = 0$, where it falls short (Fig. 3) in comparison to experiment. This is not surprising, as we have encountered this deficit already in the photoproduction of pions [25].

We now come to the axial-vector form factors. Kitagaki *et al.* [18] assume the Adler form for the dependence on Q^2, and then try to fit their new data to a range of M_A. Thus, they use

$$C_i^A(Q^2) = \frac{c_i(0)\left(1 + a_i Q^2/(b_i + Q^2)\right)}{\left(1 + Q^2/M_A^2\right)^2},\ i = 3, 4, 5, \tag{96}$$

where a's b's and c's are all model-dependent parameters determined in the Adler model [4]:

$$\begin{gathered} c_3(0) = a_3 = b_3 = 0, \\ c_4(0) = -0.3, \\ c_5(0) = 1.2, \\ a_4 = a_5 = -1.21, \\ b_4 = b_5 = 2. \end{gathered} \tag{97}$$

By fitting M_V and M_A simultaneously to their data, Kitagaki *et al.* get, with M_V given Eq.(93):

$$M_A = 0.97^{+0.14}_{-0.11}\, GeV. \tag{98}$$

Their results are found to be consistent with other earlier experiments and analysis. We thus compare our QM results with the above parameterization, where the experimental information controls the value of M_A. Our agreement with the empirical forms needed by the data of Kitagaki *et al.* is satisfactory in the IK model (Figs. 3, 9).

Given our theoretical difficulties in computing the longitudinal structure functions in the QM, we shall compare our calculations of the axial-vector helicity amplitudes, $A^A_{3/2}$ and $A^A_{1/2}$, with the experiments via the form factors C_3^A, C_4^A, C_5^A. In Figs. 7-9, we display this comparison.

Some cautionary remarks are in order regarding the quality of comparison of the IK (or any other) quark model and the "experimental values" of the Adler form factors [4] that we have displayed in Figs. 3-10. The analysis of the latest experimental results on neutrino scattering by Kitagaki *et al.* [18] does not separate the individual form factors directly by experiment. Also, it makes strong assumptions on the Q^2 dependence of the nucleon the Delta form factors, basically following the polynomial forms prescribed by Adler [4]. Finally, it neglects the non-resonant Born contributions [25], which may be small, but uninvestigated at this time.

Below we give numerical expressions of the helicity amplitudes and the Adler amplitudes in the IK model. Since the differences between $A^{V,A}_{3/2}$ and $\sqrt{3}A^{V,A}_{3/2}$ are small, we give $M1$ and $(M1)^A$ instead:

$$M1 = \frac{1}{\sqrt{3}} \frac{k}{2m} A(\overline{k}) \left(2.0237 + 0.0623\overline{k}^2 - 0.00133\overline{k}^4\right), \tag{99}$$

$$E2 = -\frac{1}{\sqrt{3}} \frac{k}{2m} A(\overline{k}) \left(6.26 + 0.627\overline{k}^2 + 0.252\overline{k}^4\right) \times 10^{-3}, \tag{100}$$

$$S^V = -\frac{1}{6\sqrt{15}} A(\overline{k})\overline{k}^2 \left(0.0996 + 0.00133\overline{k}^2\right), \tag{101}$$

$$L^V = \sqrt{3}A(\overline{k})\frac{k}{m}\left(-0.00104 + 0.000411\overline{k}^2\right), \tag{102}$$

$$(M1)^A = \frac{1}{\sqrt{3}}A(\overline{k})\left(2.0204 + 0.0625\overline{k}^2 - 0.00133\overline{k}^4\right), \tag{103}$$

$$(E2)^A = \frac{1}{\sqrt{3}}A(\overline{k})\left(0.325\overline{k}^2 - 0.251\overline{k}^4\right) \times 10^{-3}, \tag{104}$$

$$L^A = -A(\overline{k})\left(0.8235 + 0.0225\overline{k}^2 - 0.000935\overline{k}^4\right), \tag{105}$$

$$S^A = \sqrt{\frac{2}{3}}\frac{k}{m}A(\overline{k})\left(0.338 + 0.00546\overline{k}^2 - 0.000205\overline{k}^4\right). \tag{106}$$

The IK model parameters have some uncertainties due to variable estimates in the baryon eigenvalues. So we present below the result of our calculation, using another set of IK model parameters, given by Gershtein and Dzhikiya [24]. The wave-functions in this case are given by

$$|N >= 0.96|^2S_s > -0.18|^2S_{s'} > -0.22|^2S_M > -0.051|^4D_M > -0.0039|^2P_A >, \tag{107}$$

$$|\Delta >= 0.98|^4S_s > +0.16|^4S_{s'} > -0.11|^4D_s > +0.088|^2D_M > . \tag{108}$$

They correspond to energy values of $M_N = 944MeV$ and $M_\Delta = 1234MeV$. The weak amplitudes are:

$$M1 = \frac{1}{\sqrt{3}}\frac{k}{2m}A(\overline{k})\left(2.1159 + 0.0564\overline{k}^2 - 0.00084\overline{k}^4\right), \tag{109}$$

$$E2 = -\frac{1}{\sqrt{3}}\frac{k}{2m}A(\overline{k})\left(10.29 + 1.03\overline{k}^2 + 0.236\overline{k}^4\right) \times 10^{-3}, \tag{110}$$

$$S^V = -\frac{1}{6\sqrt{15}}A(\overline{k})\overline{k}^2\left(0.1330 + 0.00171\overline{k}^2\right), \tag{111}$$

$$L^V = \sqrt{3}A(\overline{k})\frac{k}{m}\left(-0.00171 + 0.000549\overline{k}^2\right), \tag{112}$$

$$(M1)^A = \frac{1}{\sqrt{3}}A(\overline{k})\left(2.1114 + 0.0567\overline{k}^2 - 0.00084\overline{k}^4\right), \tag{113}$$

$$(E2)^A = \frac{1}{\sqrt{3}}A(\overline{k})\left(0.0521\overline{k}^2 - 0.236\overline{k}^4\right) \times 10^{-3}, \tag{114}$$

$$L^A = -A(\overline{k})\left(0.862 + 0.0189\overline{k}^2 - 0.000737\overline{k}^4\right), \tag{115}$$

$$S^A = \sqrt{\frac{2}{3}}\frac{k}{m}A(\overline{k})\left(0.337 + 0.00480\overline{k}^2 - 0.000172\overline{k}^4\right). \tag{116}$$

This gives us a feeling of the sensitivity of the weak helicity amplitudes to small changes in the IK wave functions. The form-factors calculated from this choice is labeled as “Isgur-Karl 2” in Figs. 1-11.

We also include the result of a D-state mixing model, suggested by Glashow [46], and further discussed by VBJ [46], Bourdeau and Mukhopadhyay [32] :

$$|N>= \sqrt{1-\gamma}|^2S_s> + \sqrt{\gamma}|^4D_M>, \tag{117}$$

$$|\Delta>= \sqrt{1-3\beta}|^4S_s> - \sqrt{2\beta}|^4D_s> + \sqrt{\beta}|^2D_M> . \tag{118}$$

The purpose here is to get the correct g_A by changing γ, and adjust β so that the EMR comes out to be the PDG recommended value. We find

$$\gamma = 0.2048 \pm 0.0015,\ \beta = 0.103 \pm 0.011, \tag{119}$$

and together they give:

$$g_A = 1.257 \pm 0.003,\ EMR = -0.015 \pm 0.004. \tag{120}$$

The results of this model is included in Table IV and Figs. 1-11.

A recent experiment [37] shows that $EMR = -0.026$. This implies, in the D-state mixing model, $\beta = 0.073$. For brevity, we do not give numerical results for the helicity amplitudes for this case.

E. The $(E2)^A\,/\,(M1)^A$ ratio

The ratio $E2/M1$ in the $\gamma N \to \Delta$ transition has attracted a lot of attention of theorists [25,36,37] and experimentalists [38] alike. In analogy to this quantity, we can also define the ratio

$$AEMR \equiv (E2)^A\,/\,(M1)^A\,, \tag{121}$$

for the axial-vector analogue of the EMR for the process (3). In the SU(6) symmetry limit, it is zero identically.

We give in Fig. 11 a plot of the ratio as a function of Q^2, keeping W fixed at the Delta excitation, as computed from the QM, and compare it with the Adler assumption that it is zero identically, an assumption made in the Kitagaki *et al.* [18] analysis. It would be interesting to determine this quantity directly from the experiment in the future. We recall here that the EMR , for the vector current, is not well-determined in the QM of the IK variety. While Davidson *et al.* [25] found the value for this ratio at the photon point to be

$$EMR\left(Q^2 = 0,\ [25]\right) = -0.0157 \pm 0.0072, \tag{122}$$

from the pion photoproduction in the Delta region. New experiments [37] at the Brookhaven LEGS facility have indicated this ratio to be at least *twice* as large. The IK quark model predicts the ratio at the photon point

$$EMR\left(Q^2 = 0; IK\ Model\right) = -0.0033. \tag{123}$$

The alternative wave-functions of IK model (Isgur-Karl 2), given above (Eq.(105-106)), yield an EMR of -0.0051. Thus, the EMR, obtained in the IK quark model, is much smaller in magnitude compared to the phenomenological value. There are indications in the latest Skyrmion approach [13] that the EMR at the photon point is indeed much larger than the prediction of the IK quark model, close to -0.05. Thus, the experimental determination of the AEMR may be less difficult than what the quark model suggests, as the latter may turn out to be a gross underestimate. This quantity should be computed in the soliton models.

While we are on the subject of the EMR and AEMR, we should make some remarks about the issue of the behavior of these ratios as functions of Q^2, a very topical question. In the perturbative QCD approach (pQCD) [40], these ratios must approach unity, as $Q^2 \to \infty$:

$$EMR\left(Q^2 \to \infty\right) \Rightarrow 1,\ AEMR\left(Q^2 \to \infty\right) \Rightarrow 1, \tag{124}$$

while in the IK model, these behave as follows:

$$EMR = -\frac{6.26 + 0.627\overline{k}^2 + 0.252\overline{k}^4}{2.0237 + 0.0623\overline{k}^2 - 0.00133\overline{k}^4} \times 10^{-3}, \tag{125}$$

$$AEMR = \frac{0.325\overline{k}^2 - 0.251\overline{k}^4}{2.0204 + 0.0625\overline{k}^2 - 0.00133\overline{k}^4} \times 10^{-3}. \tag{126}$$

The last two equations are, of course, meaningless at large Q^2 (that is, large $\overline{k}^2$). We are giving the last two equations merely to indicate very different limits for the EMR and AEMR, at large Q^2, in the IK quark model, compared with pQCD. Nevertheless, it is amusing to note that the IK quark model reproduces the signs and rough orders of magnitudes of the pQCD limits mentioned above.

Several questions are important here: (1) Do the EMR and AEMR approach unity for large Q^2 attainable in the laboratory? If the answer is yes, at what Q^2? (2) Where does the IK-type quark model predictions definitely break down? (3) What happens to the Bloom-Gilman duality [41] in the axial vector sector, on which we have no experimental information? Future research should focus on these issues. We may add here that there is a strong debate in the literature on the question (1). The pQCD "believers" [40] tend to see the asymptopia at hand at $Q^2 \geq 6\,GeV^2$, while the "non-believers" [42] argue that the Q^2 value for the pQCD rules to be valid is too large for us to worry, at the "modest" Q^2 currently available.

F. Further comments on the comparison of our work with others

Previous to our work, many authors have investigated the neutrino excitation of the Delta resonance off nucleons, but all in the context of the SU(6) symmetric quark model [5–8]. Schreiner and von Hippel [10] have reviewed most of these attempts. Among the later works, mention may be made of the work of Abdullah and Close [8]. In this work, the CVC condition is imposed, also, quark structure functions are introduced, and the quark form factors are treated as constants. In the work by Andreadis *et al.* [11], again in the SU(6) limit, quark form factors are introduced. We do not use form factors at the quark level for the following reason: (1) we want to test quark model predictions rather than fitting a constituent-quark form-factor. (2) We believe that the effects of the gluons and sea-quarks have already been absorbed in the effective potential of Isgur-Karl quark model. (3) Our results *without* the form-factor is very good compared to the currently available experiments. Thus, there is no phenomenological reason to introduce quark form factors. (4) Introduction of quark-level form factors would complicate even more the relationship of the quark model to QCD. As it is, this relationship is far from clear.

Finally, we should discuss the difference between our approach here and a recent work by Hemmert, Holstein and Mukhopadhyay(HHM) [14] dealing with the NN and NΔ couplings in the quark model (see Table IV). In that work, relativistic corrections to the nucleon g_A are taken into account and an agreement with the experimental g_A for the nucleon is reached. But it yields an off-diagonal NΔ axial coupling substantially lower than the experimental values. In contrast to HHM, the non-relativistic IK approach, used here, does not have the relativistic correction taken into account. Thus, the diagonal value of g_A still remains off the experimental value ($g_A(IK) = 1.63$). But the off-diagonal axial vector matrix elements come out better. Overall, *the color hyperfine interaction does not remove the discrepancy between quark model estimates and experiments in either approach, when both diagonal and off-diagonal effects are computed*. Thus, the problem of the quark model in simultaneously explaining diagonal *and* off-diagonal observables does not disappear.

VII. SUMMARY AND CONCLUSIONS

We have computed weak amplitudes of the $N \to \Delta$ transition in the framework of the IK quark model, thereby dealing with the effect of the color-hyperfine interactions in these amplitudes. Our main conclusions are:

1. The deficit of the nucleon to Delta (1232) magnetic dipole amplitude in the IK quark model estimate, when compared with pion photoproduction analysis [25], is confirmed via the vector form factor C_3^V at $q^2 \to 0$. This deficit seems to heal around of $Q^2 = 0.25$ GeV^2.

2. There is a mild violation of the CVC in the IK model. Thus, the amplitude C_6^V , which should be zero by CVC, is predicted in the IK quark model to have a small but non-zero value.

3. The axial-vector transverse amplitudes are largely well-described in the IK model. An exception is our inability to get the PCAC value of the C_6^A. This is not surprising, since we do not have explicit meson degrees of freedom. The IK model also violates axial current conservation in the chiral limit. Despite this shortcoming, we are able to reproduce the off-diagonal Goldberger-Treiman estimate of the $C_5^A(0)$ in the IK quark model.

4. There is a simple way of parameterizing the SU(6) breaking effects, through a relation (Eq.(56)) that connects the two transverse helicity amplitudes to the Adler form factor C_3^A. The IK model gives an estimate of this small effect. Its experimental verification, though indirect and difficult, should constitute an important experimental challenge, analogous to the determination of E2/M1 in the vector (electromagnetic) sector.

Hopefully, new weak interactions studies in the nucleon resonance region (1-2 GeV of W) will be possible at existing neutrino facilities. There are also new experimental possibilities at the CEBAF on the weak charged and neutral current explorations [43-47] of the isobar physics. Though these are intrinsically very difficult, there is some hope that such experiments would be possible. They would go a long way towards our understanding of the axial vector response of the nucleon, in particular. The role of the Delta isobar, already important in the Adler-Weissberger sum-rule [48], would be interesting to be explored further in the nucleon to Delta weak excitation domain.

Acknowledgments

One of us (NCM) thanks T. Hemmert, B. Holstein and J. Napolitano for many helpful discussions. This research has been supported by the U.S. Dept. of Energy.

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	$\sqrt{\frac{3}{2}}L^A_{1/2}$	$\sqrt{\frac{3}{2}}\frac{3m}{k}S^A_{1/2}$
$<\Delta^4 S_s\|N^2 S_s>$	$-\frac{2}{\sqrt{3}}$	$\frac{1}{3\sqrt{3}}$
$<\Delta^4 S_s\|N^2 S_{s'}>$	$\frac{1}{9}\overline{k}^2$	$-\frac{2}{9}\left(1+\frac{1}{12}\overline{k}^2\right)$
$<\Delta^4 S_{s'}\|N^2 S_s>$	$\frac{1}{9}\overline{k}^2$	$\frac{2}{9}\left(1-\frac{1}{12}\overline{k}^2\right)$
$<\Delta^4 S_s\|N^2 S_M>$	$\frac{1}{9\sqrt{2}}\overline{k}^2$	$-\frac{\sqrt{2}}{9}\left(1+\frac{1}{12}\overline{k}^2\right)$
$<\Delta^4 S_s\|N^4 D_M>$	$\frac{1}{18\sqrt{5}}\overline{k}^2$	$-\frac{1}{6\sqrt{5}}-\frac{1}{9\sqrt{5}}\left(1+\frac{1}{12}\overline{k}^2\right)$
$<\Delta^4 D_s\|N^2 S_s>$	$-\frac{1}{9}\sqrt{\frac{2}{5}}\overline{k}^2$	$-\frac{1}{3}\sqrt{\frac{2}{5}}-\frac{2}{9}\sqrt{\frac{2}{5}}\left(1-\frac{1}{12}\overline{k}^2\right)$
$<\Delta^2 D_M\|N^2 S_s>$	$\frac{1}{18\sqrt{5}}\overline{k}^2$	$\frac{1}{6\sqrt{5}}+\frac{1}{9\sqrt{5}}\left(1-\frac{1}{12}\overline{k}^2\right)$
$<\Delta^4 S_{s'}\|N^2 S_{s'}>$	$\frac{1}{6\sqrt{3}}\left(-12+\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$\frac{1}{3\sqrt{3}}\left(1-\frac{1}{9}\overline{k}^2+\frac{1}{108}\overline{k}^4\right)$
$<\Delta^4 S_{s'}\|N^2 S_M>$	$\frac{1}{6\sqrt{6}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$-\frac{1}{27\sqrt{6}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$
$<\Delta^4 S_{s'}\|N^4 D_M>$	$\frac{1}{9\sqrt{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$	$\frac{1}{36\sqrt{15}}\overline{k}^2-\frac{1}{54\sqrt{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$
$<\Delta^4 D_s\|N^2 S_{s'}>$	$-\frac{2}{9}\sqrt{\frac{2}{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$	$\frac{1}{18}\sqrt{\frac{2}{15}}\overline{k}^2+\frac{1}{27}\sqrt{\frac{2}{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$
$<\Delta^4 D_s\|N^2 S_M>$	$-\frac{2}{9\sqrt{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$	$\frac{1}{18\sqrt{15}}\overline{k}^2+\frac{1}{27\sqrt{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$
$<\Delta^4 D_s\|N^4 D_M>$	$-\frac{1}{5\times 18\sqrt{6}}\left(13\overline{k}^2-\frac{1}{3}\overline{k}^4\right)$	$\frac{\sqrt{6}}{12}\left(1-\frac{1}{15}\overline{k}^2\right)+\frac{1}{540\sqrt{6}}\left(13\overline{k}^2-\frac{1}{3}\overline{k}^4\right)$
$<\Delta^2 D_M\|N^2 S_{s'}>$	$\frac{1}{9\sqrt{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$	$-\frac{1}{36\sqrt{15}}\overline{k}^2-\frac{1}{54\sqrt{15}}\left(\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$
$<\Delta^2 D_M\|N^2 S_M>$	$\frac{1}{12\sqrt{30}}\left(\frac{16}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$-\frac{1}{36\sqrt{30}}\overline{k}^2-\frac{1}{54\sqrt{30}}\left(4\overline{k}^2-\frac{1}{12}\overline{k}^4\right)$
$<\Delta^2 D_M\|N^4 D_M>$	$\frac{1}{30\sqrt{3}}\left(30-\frac{17}{3}\overline{k}^2+\frac{2}{9}\overline{k}^4\right)$	$\frac{1}{2\sqrt{3}}\left(1-\frac{1}{15}\overline{k}^2\right)-\frac{1}{54\sqrt{3}}\left(9-\frac{17}{10}\overline{k}^2+\frac{1}{15}\overline{k}^4\right)$

TABLE I. Axial-vector longitudinal and scalar amplitudes for various nucleon to Delta SU(6) configurations. Common factor is $\sqrt{\frac{4\pi\alpha_W}{2K_0}}e^{-\overline{k}^2/6}$.

	$\sqrt{3}A^A_{1/2}$	$A^A_{3/2}$
$<\Delta^4 S_s\|N^2 S_s>$	$-\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$
$<\Delta^4 S_s\|N^2 S_{s'}>$	$\frac{1}{9}\overline{k}^2$	$\frac{1}{9}\overline{k}^2$
$<\Delta^4 S_{s'}\|N^2 S_s>$	$\frac{1}{9}\overline{k}^2$	$\frac{1}{9}\overline{k}^2$
$<\Delta^4 S_s\|N^2 S_M>$	$\frac{1}{9\sqrt{2}}\overline{k}^2$	$\frac{1}{9\sqrt{2}}\overline{k}^2$
$<\Delta^4 S_s\|N^4 D_M>$	$\frac{2}{9\sqrt{5}}$	$-\frac{1}{9\sqrt{5}}\overline{k}^2$
$<\Delta^4 D_s\|N^2 S_s>$	$-\frac{2}{9\sqrt{10}}\overline{k}^2$	$\frac{2}{9\sqrt{10}}\overline{k}^2$
$<\Delta^2 D_M\|N^2 S_s>$	$-\frac{1}{9\sqrt{5}}\overline{k}^2$	0
$<\Delta^4 S_{s'}\|N^2 S_{s'}>$	$\frac{1}{6\sqrt{3}}\left(-12+\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$\frac{1}{6\sqrt{3}}\left(-12+\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$
$<\Delta^4 S_{s'}\|N^2 S_M>$	$\frac{1}{6\sqrt{6}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$\frac{1}{6\sqrt{6}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$
$<\Delta^4 S_{s'}\|N^4 D_M>$	$\frac{1}{3\sqrt{15}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$\frac{1}{6\sqrt{15}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$
$<\Delta^4 D_s\|N^2 S_{s'}>$	$-\frac{1}{3\sqrt{30}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$\frac{1}{3\sqrt{30}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$
$<\Delta^4 D_s\|N^2 S_M>$	$-\frac{1}{6\sqrt{15}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	$\frac{1}{6\sqrt{15}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$
$<\Delta^4 D_s\|N^4 D_M>$	$-\frac{1}{15\sqrt{6}}\left(\frac{17}{3}\overline{k}^2-\frac{2}{9}\overline{k}^4\right)$	$-\frac{1}{15\sqrt{6}}\left(\frac{10}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$
$<\Delta^2 D_M\|N^2 S_{s'}>$	$-\frac{1}{6\sqrt{15}}\left(\frac{4}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	0
$<\Delta^2 D_M\|N^2 S_M>$	$-\frac{1}{6\sqrt{30}}\left(\frac{16}{3}\overline{k}^2-\frac{1}{9}\overline{k}^4\right)$	0
$<\Delta^2 D_M\|N^4 D_M>$	$\frac{1}{30\sqrt{3}}\left(30-\frac{17}{3}\overline{k}^2+\frac{2}{9}\overline{k}^4\right)$	$\frac{1}{\sqrt{3}}\left(1-\frac{1}{30}\overline{k}^2\right)$

TABLE II. Transverse axial-vector amplitudes for various nucleon to Delta SU(6) configurations. Common factor is $\sqrt{\frac{4\pi\alpha_W}{2K_0}}e^{-\overline{k}^2/6}$.

	$\frac{m}{k}L^V_{1/2}$	$S^V_{1/2}$	$\sqrt{3}A^V_{1/2}(H^V_{II})$	$A^V_{3/2}(H^V_{II})$
$<\Delta^2 D_M\|N^2 S_s>$	$-\frac{1}{3\sqrt{15}}\left(1-\frac{\overline{k}^2}{12}\right)$	$\frac{1}{6\sqrt{15}}\overline{k}^2$	$\frac{1}{\sqrt{5}}$	$-\frac{1}{3\sqrt{5}}$
$<\Delta^2 D_M\|N^2 S_{s'}>$	$\frac{1}{54\sqrt{5}}\left(\overline{k}^2-\frac{\overline{k}^4}{12}\right)$	$\frac{1}{9\sqrt{5}}\left(\overline{k}^2-\frac{\overline{k}^4}{12}\right)$	$-\frac{1}{6\sqrt{15}}\overline{k}^2$	$\frac{1}{18\sqrt{15}}\overline{k}^2$
$<\Delta^2 D_M\|N^2 S_M>$	$-\frac{1}{648\sqrt{10}}\overline{k}^4$	$-\frac{1}{108\sqrt{10}}\overline{k}^4$	$-\frac{1}{6\sqrt{30}}\overline{k}^2$	$\frac{1}{18\sqrt{30}}\overline{k}^2$
$<\Delta^4 S_s\|N^4 D_M>$	$-\frac{1}{3\sqrt{15}}\left(1+\frac{\overline{k}^2}{12}\right)$	$-\frac{1}{6\sqrt{15}}\overline{k}^2$	$\frac{1}{\sqrt{5}}$	$-\frac{1}{3\sqrt{5}}$
$<\Delta^4 S_{s'}\|N^4 D_M>$	$-\frac{1}{54\sqrt{5}}\left(\overline{k}^2-\frac{\overline{k}^4}{12}\right)$	$-\frac{1}{9\sqrt{5}}\left(\overline{k}^2-\frac{\overline{k}^4}{12}\right)$	$-\frac{1}{6\sqrt{15}}\overline{k}^2$	$\frac{1}{18\sqrt{15}}\overline{k}^2$
$<\Delta^4 D_s\|N^4 D_M>$	$\frac{\sqrt{2}}{1080}\left(7\overline{k}^2-\frac{\overline{k}^4}{3}\right)$	$\frac{\sqrt{2}}{180}\left(7\overline{k}^2-\frac{\overline{k}^4}{3}\right)$	$-\frac{1}{\sqrt{6}}\left(1-\frac{\overline{k}^2}{15}\right)$	$-\frac{1}{\sqrt{6}}\left(1-\frac{\overline{k}^2}{15}\right)$

TABLE III. Vector amplitudes amplitudes for various nucleon to Delta SU(6) configurations. Common factor is $\sqrt{\frac{4\pi\alpha_W}{2K_0}}e^{-\overline{k}^2/6}$. The transverse contribution listed is from H^V_2 only. Transverse contribution from H^V_1 equals to its axial counterpart (Table 2) times $k/2m$. Contributions from the other configurations not shown are zero.

	C_3^V	C_4^V	C_5^V	C_6^V	C_3^A	C_4^A	C_5^A	$C^A_{6,non-pole}$
Salin [3]	2.0	0	0	0	0	-2.7	0	0
Adler [4]	1.85	-0.89	0	0	0	-0.3	1.2	0
Bijtebier [5]	2.0	0	0	0	0	-2.9∼ -3.6	1.2	0
Zucker [7]	-	-	-	0	1.8	1.8	1.9	0
HHM [14]	1.39	-	-	0	0	-0.29± 0.006	0.87± 0.03	-
SU(6)	1.48	-1.13	0	0	0	-0.38	1.17	0
Isgur-Karl	1.32	-0.79	-0.36	0.014	-0.0013	-0.66	1.16	0.032
Isgur-Karl 2	1.37	-0.66	-0.59	-0.015	0.0008	-0.657	1.20	0.042
D-mixing [32,36]	1.29	0.78	-1.9	-0.15	0.052	0.052	0.813	-0.17

TABLE IV. A comparison of the Adler form factors in the $N \to \Delta$ weak transition in different approaches indicated in the text. The momentum transfer squared is taken to be zero. The last four rows are from this work.

The Figure Captions

Figure 1: The ratio $A^V_{3/2}/\sqrt{3}A^V_{1/2}$ vs. Q^2.

Figure 2: The ratio $A^A_{3/2}/\sqrt{3}A^A_{1/2}$ vs. Q^2.

Figure 3: C^V_3 as a function of Q^2. Results from the two versions of the IK quark-model, SU(6) limit and the D-wave mixing model, are compared with the experimetal results of Kitagaki *et al* [18]. The experimental errors shown here are due to the uncertainty in M_V (Eq.(93)).

Figure 4: C^V_4 as a function of Q^2.

Figure 5: C^V_5 vs. Q^2. This form-factor is assumed to be zero in experimental fits. Its non-zero value here indicates the violation of the *magnetic dipole dominance.*

Figure 6: C^V_6 vs. Q^2. Its non-zero value indicates the degree of CVC violation in the quark model.

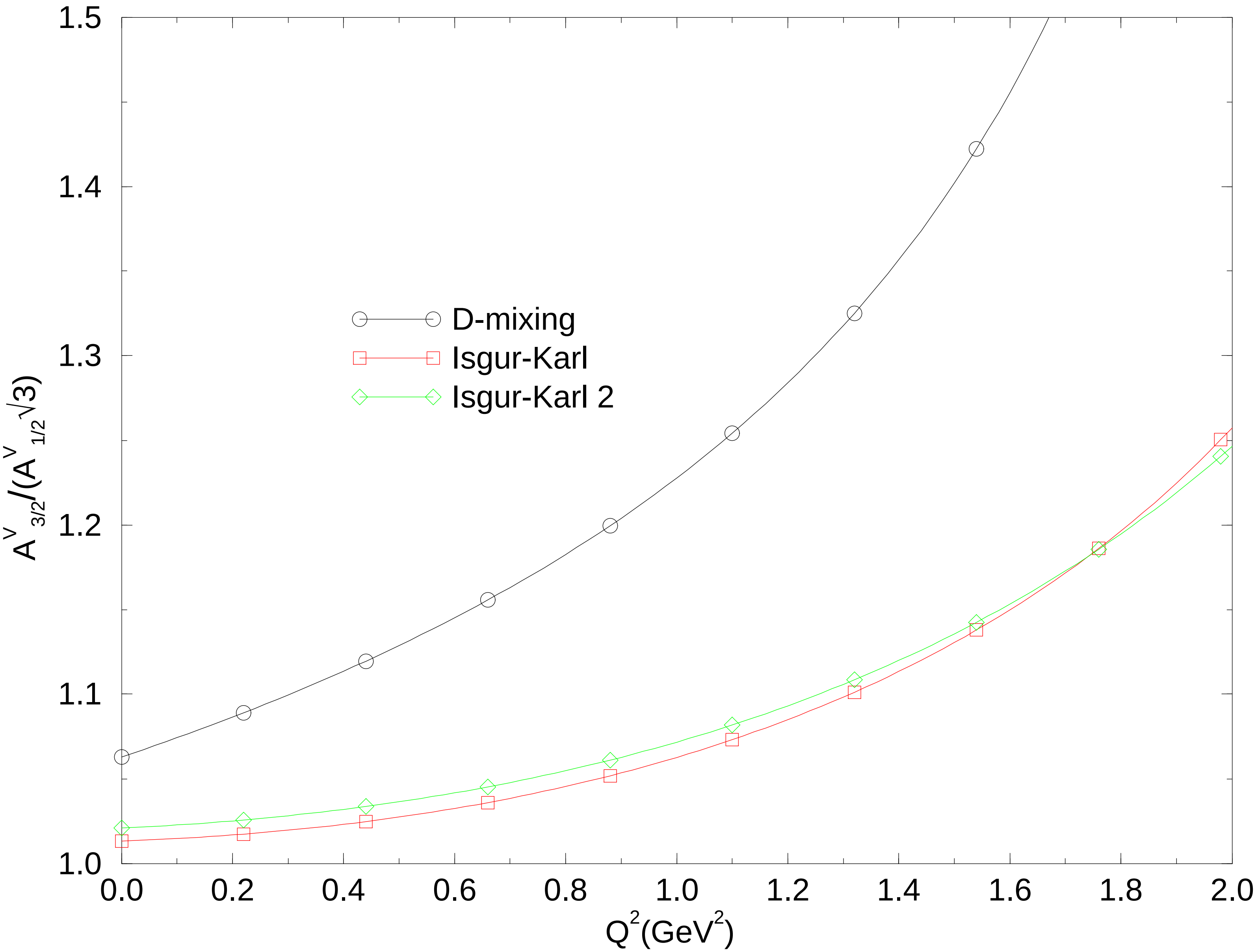
Figure 7: C^A_3 vs. Q^2. Its non-zero value indicates the violation of *magnetic dipole dominance.*

Figure 8: C^A_4 vs. Q^2.

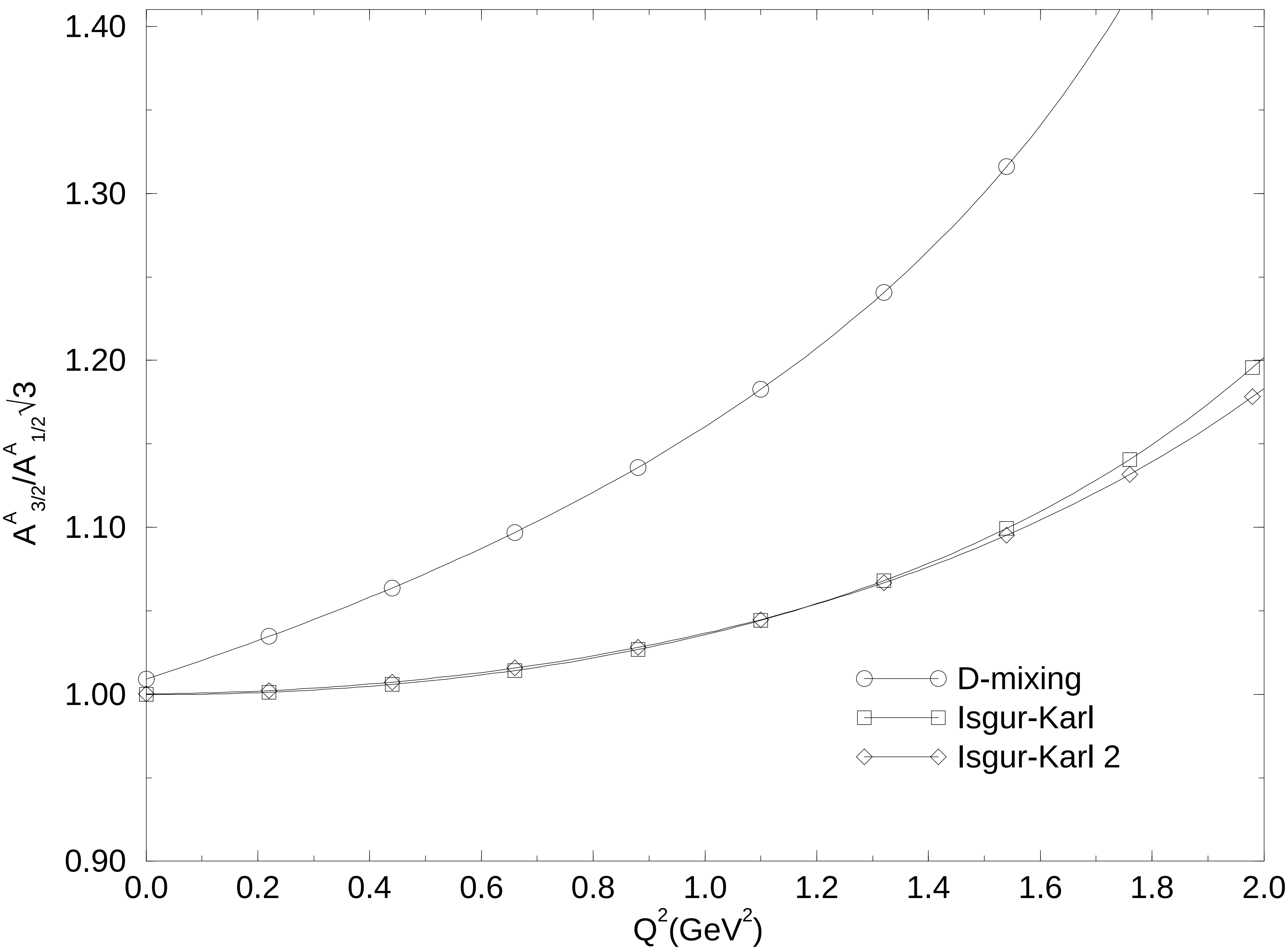
Figure 9: C^A_5 vs. Q^2. Results from two versions of the IK quark-model, SU(6) limit and D-wave mixing model are compared with experimental results of Kitagaki *et al.* The experimental errors shown are generated by the uncertainty in the fitted parameter M_A (Eq.(98)).

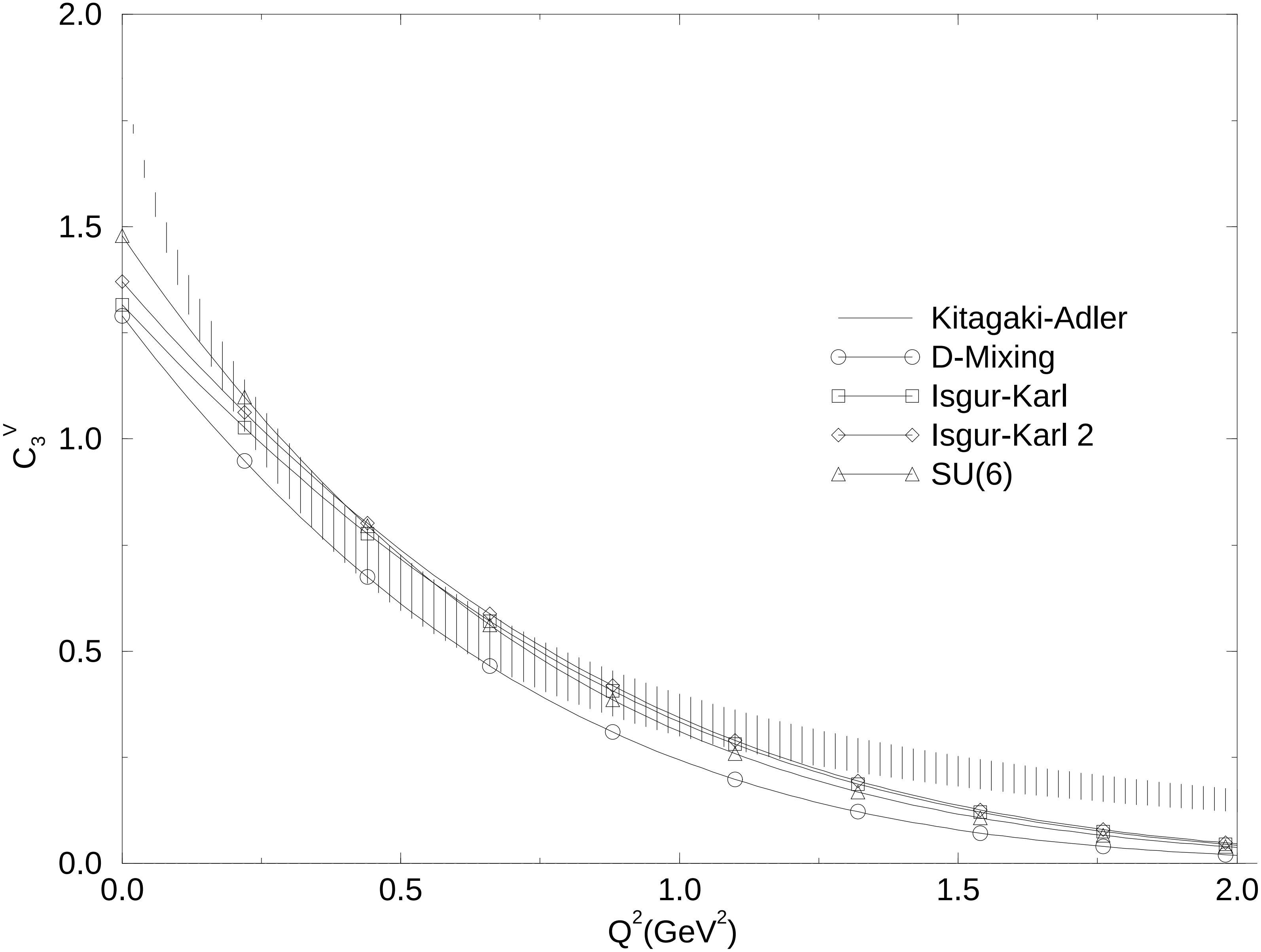
Figure 10: C^A_6 vs. Q^2. Since the quark model does not have pion-pole term built in it, this is only the non-pole contribution.

Figure 11: $E2/M1$ and $(E2)^A/(M1)^A$, EMR and AEMR respectively, vs. Q^2.

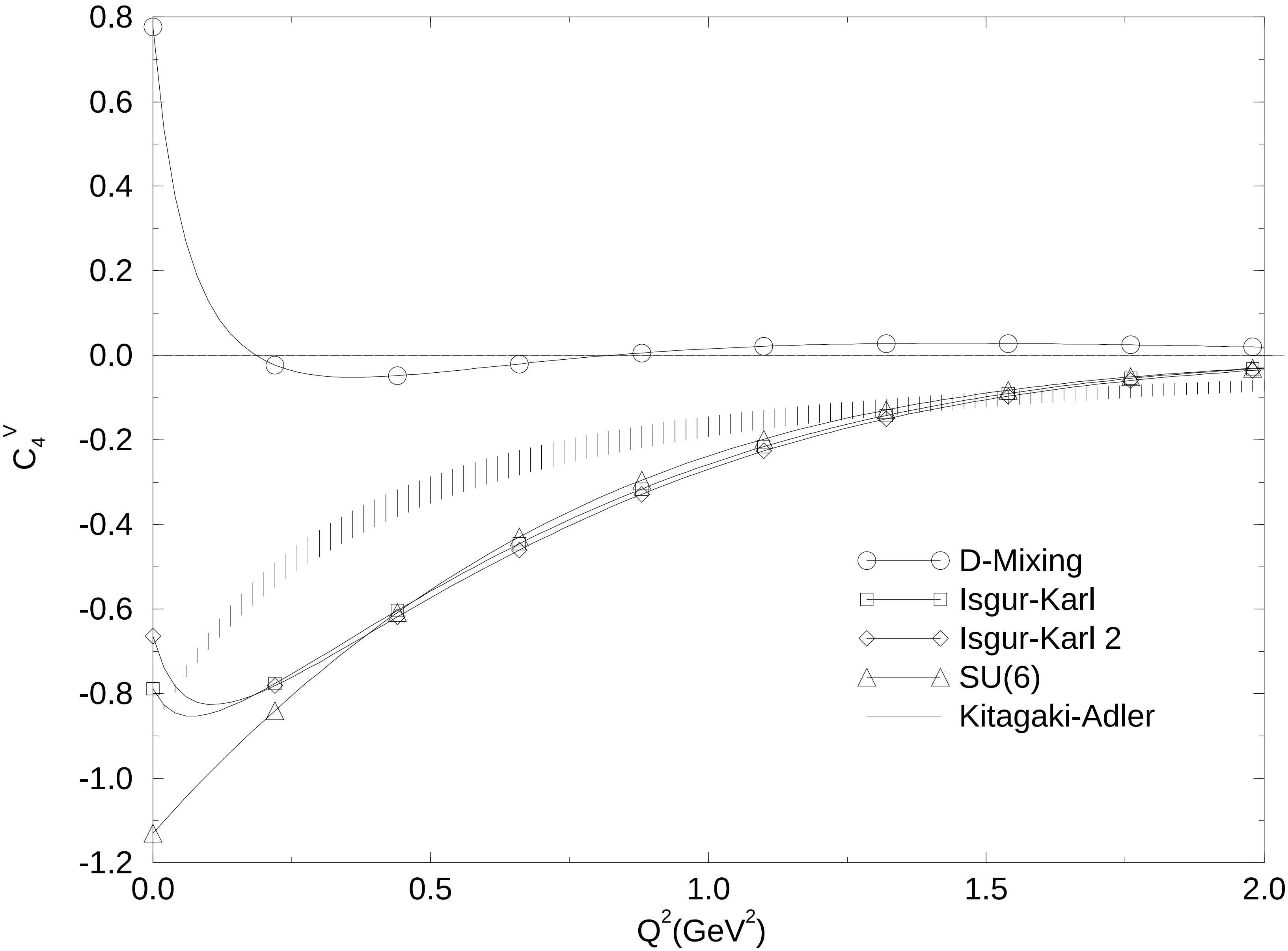


D-mixing
Isgur-Karl
Isgur-Karl 2
1.0
1.1
1.2
1.3
1.4
1.5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
$Q^2(GeV^2)$
$A^v_{3/2}/(A^v_{1/2}\sqrt{3})$

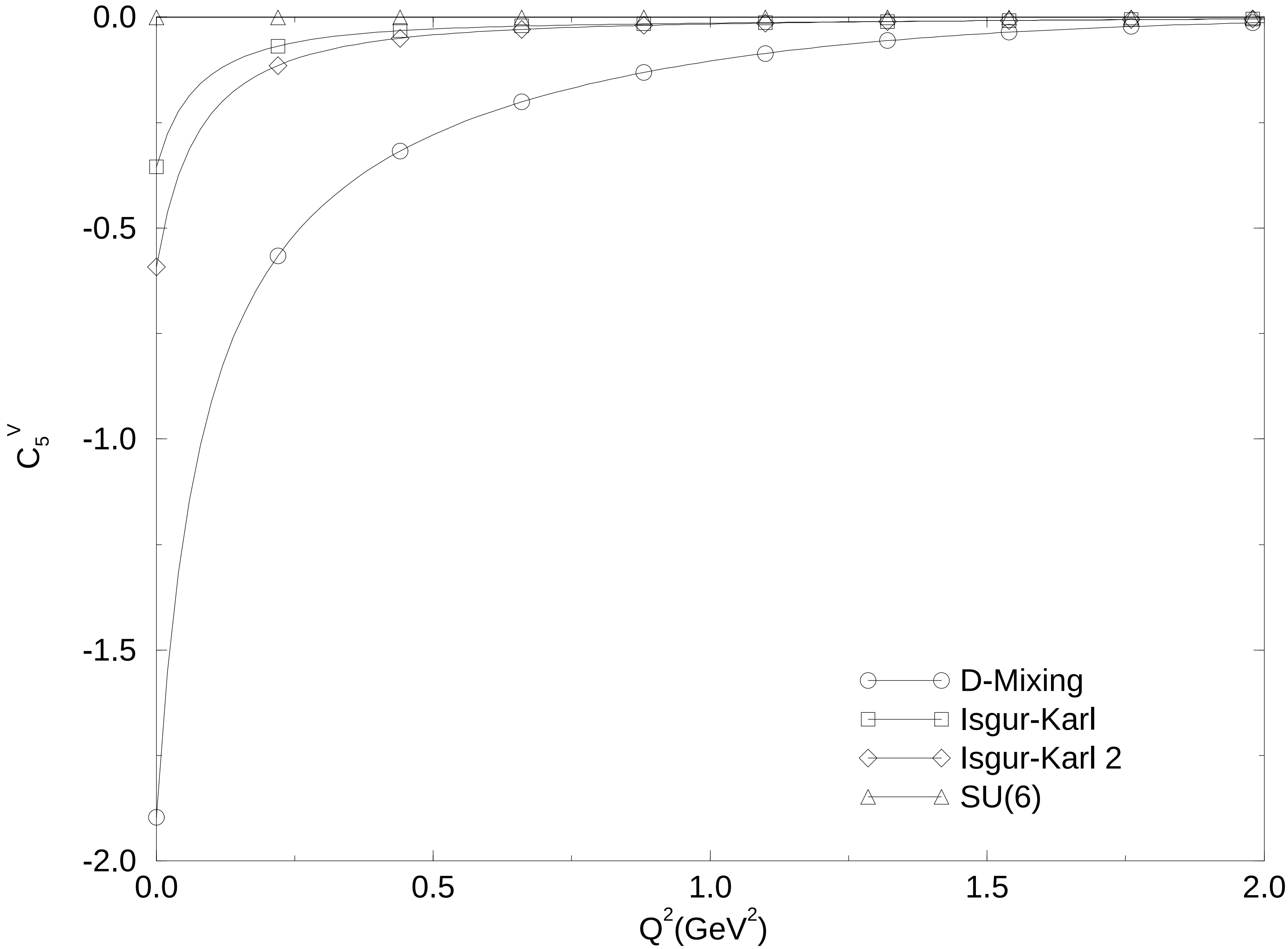

1.40
1.30
1.20
1.10
1.00
0.90
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
$A^A_{3/2}/A^A_{1/2}\sqrt{3}$
$Q^2(GeV^2)$
D-mixing
Isgur-Karl
Isgur-Karl 2



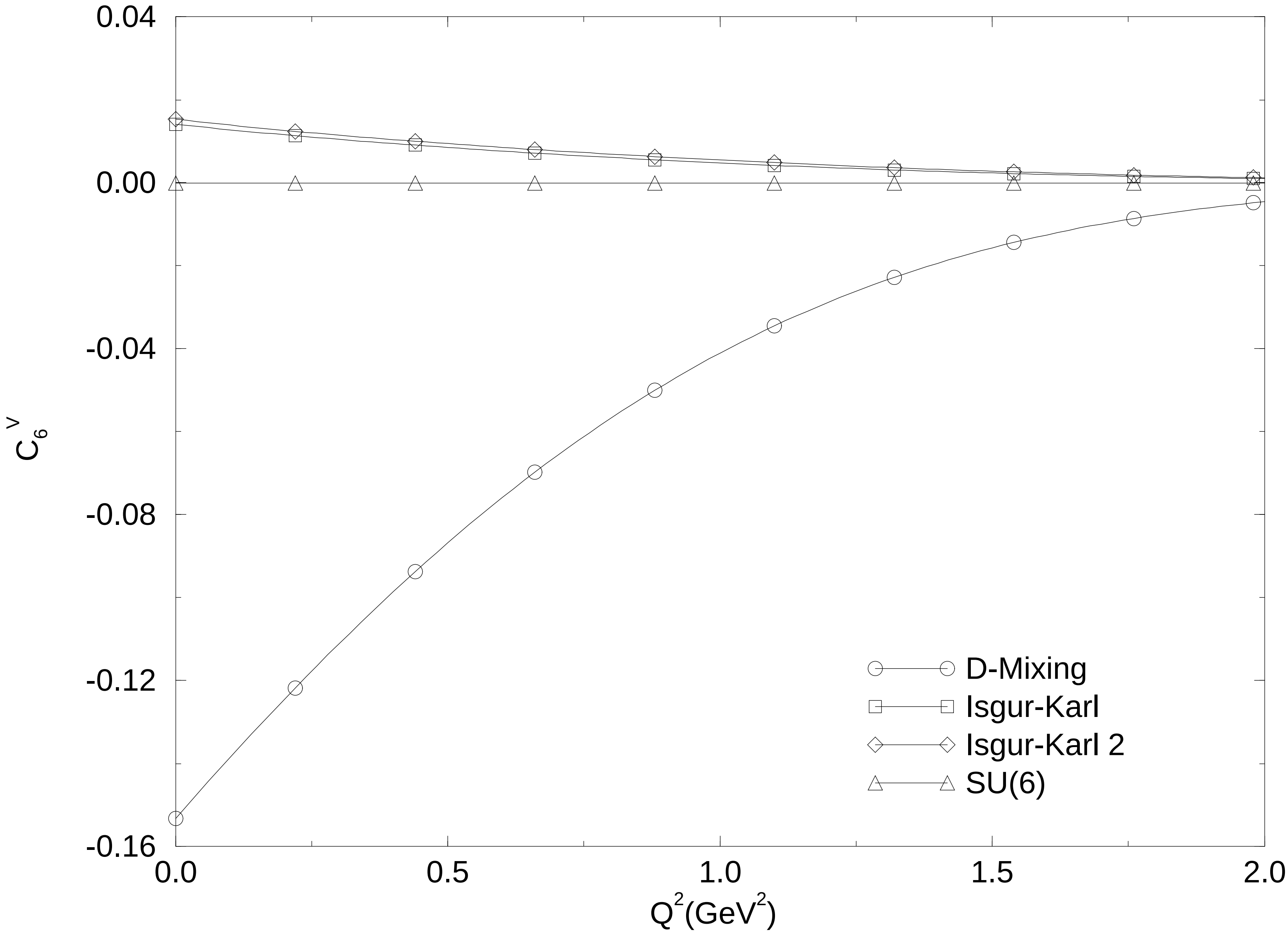
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
C_3^V
$Q^2(GeV^2)$
Kitagaki-Adler
D-Mixing
Isgur-Karl
Isgur-Karl 2
SU(6)

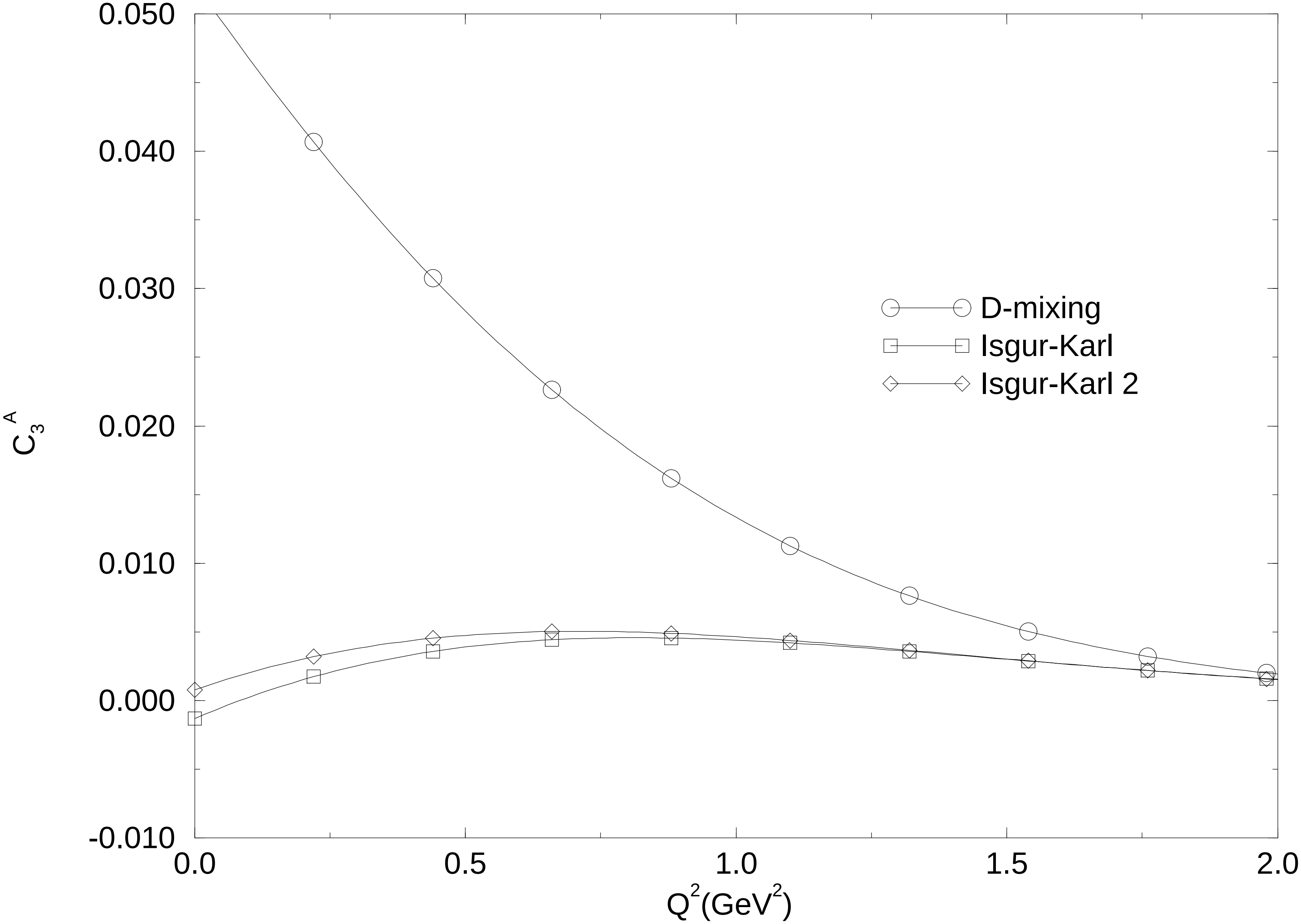


C_4^V
$Q^2(GeV^2)$
D-Mixing
Isgur-Karl
Isgur-Karl 2
SU(6)
Kitagaki-Adler

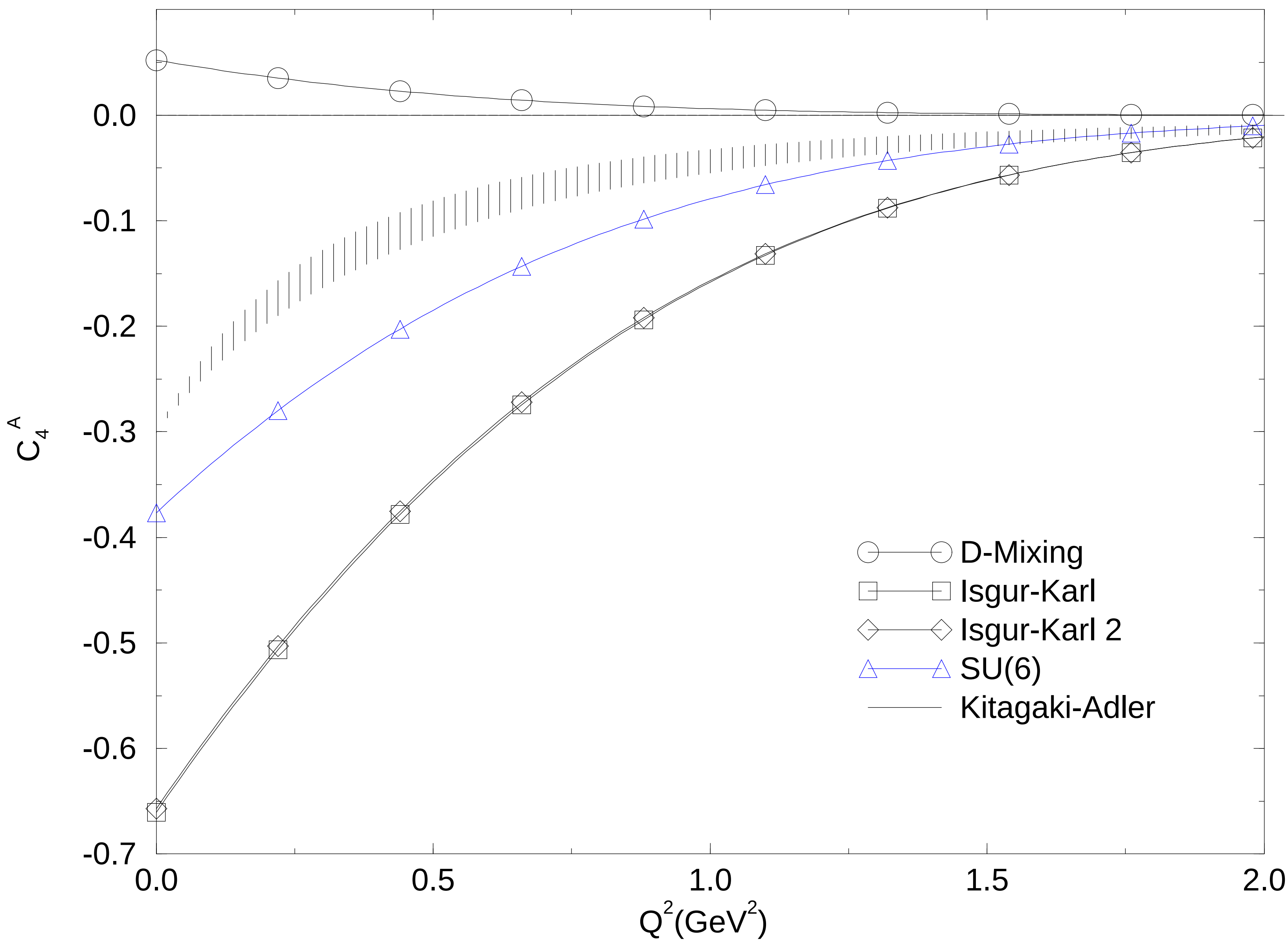


0.0
-0.5
-1.0
-1.5
-2.0
C_5^V
0.0
0.5
1.0
1.5
2.0
$Q^2(GeV^2)$
D-Mixing
Isgur-Karl
Isgur-Karl 2
SU(6)

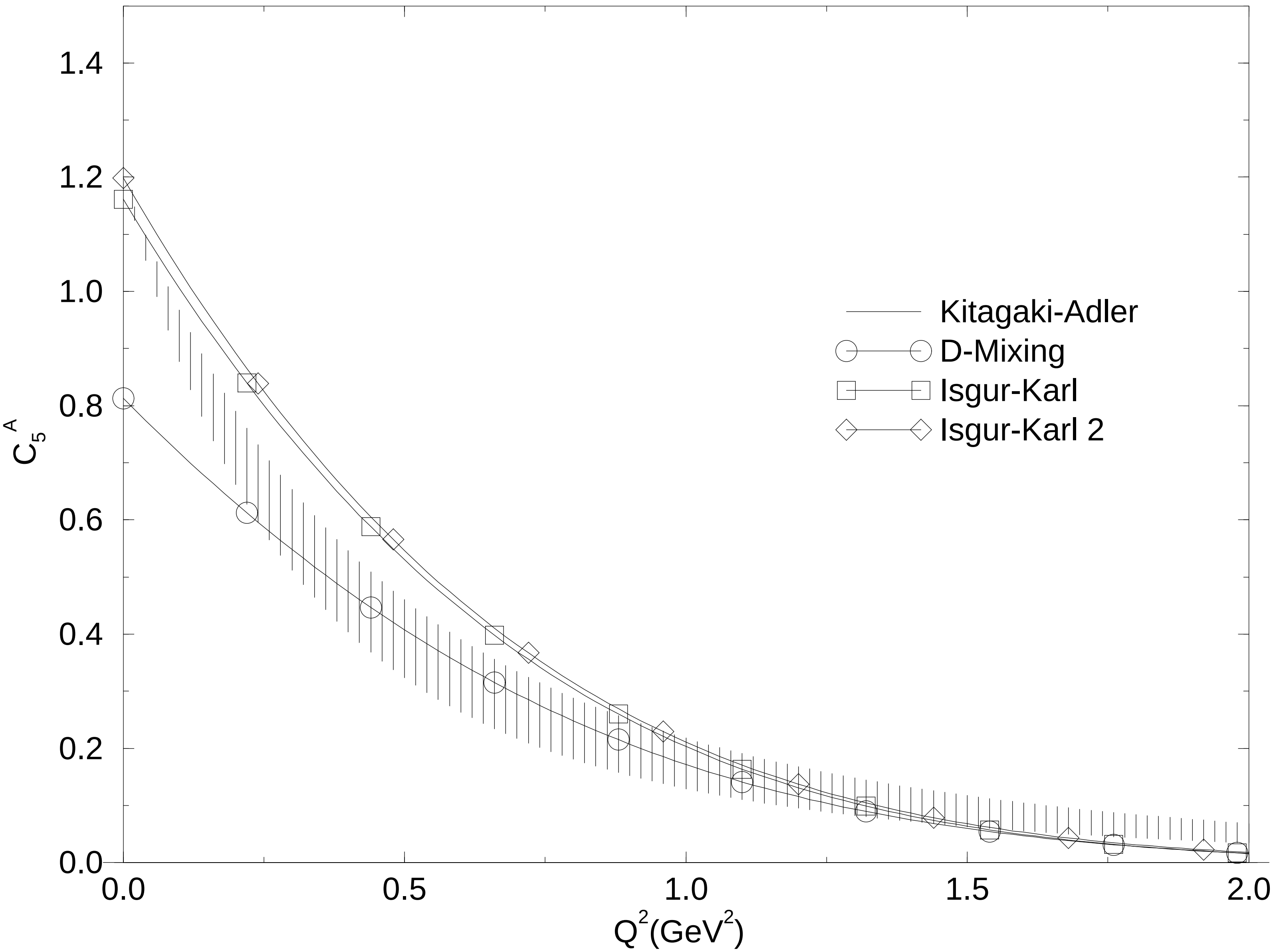

0.04
0.00
-0.04
-0.08
-0.12
-0.16
0.0
0.5
1.0
1.5
2.0
$Q^2(GeV^2)$
C_6^v
D-Mixing
Isgur-Karl
Isgur-Karl 2
SU(6)

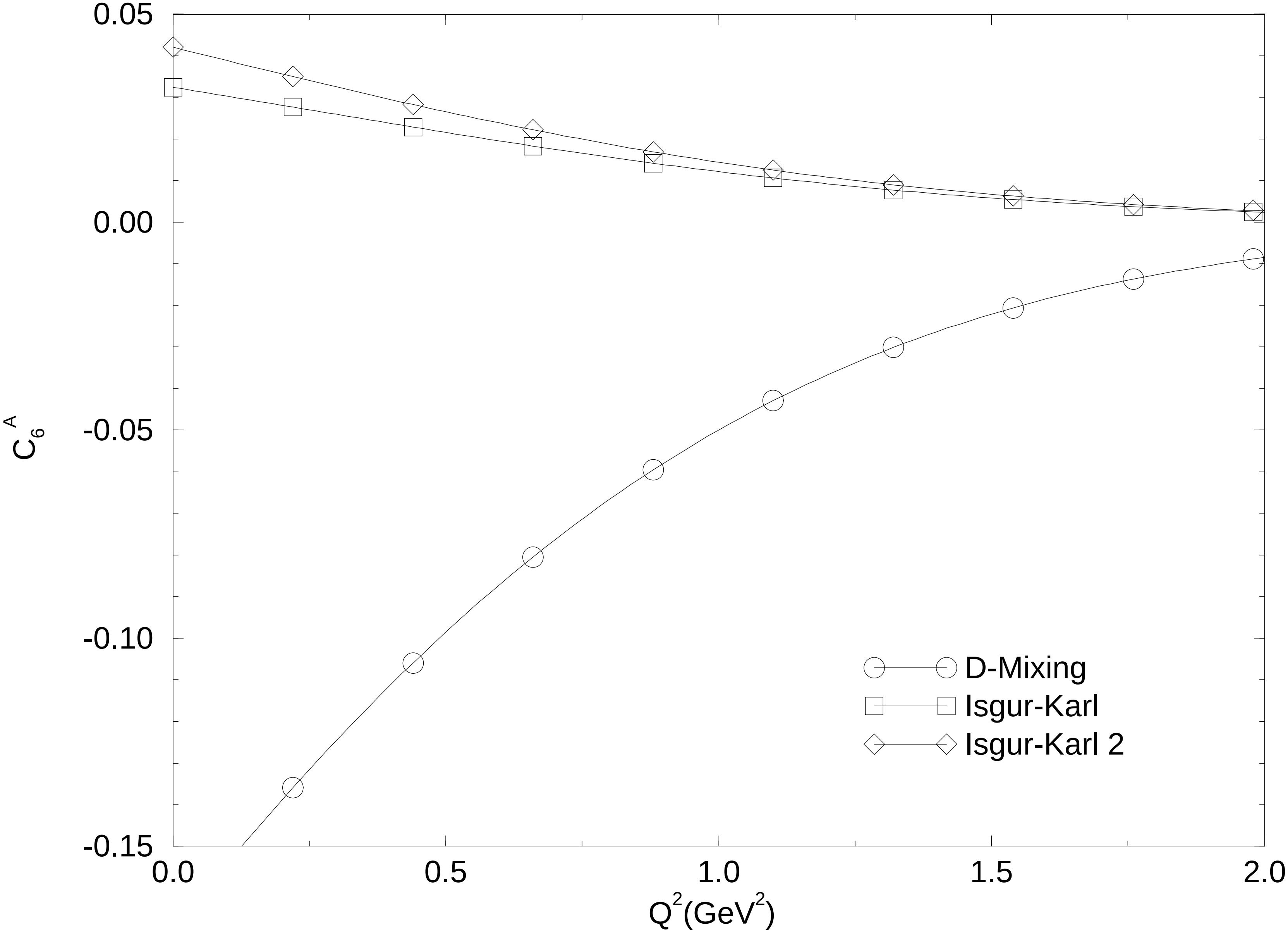


0.050
0.040
0.030
0.020
0.010
0.000
-0.010
0.0
0.5
1.0
1.5
2.0
Q2(GeV2)
C3A
D-mixing
Isgur-Karl
Isgur-Karl 2

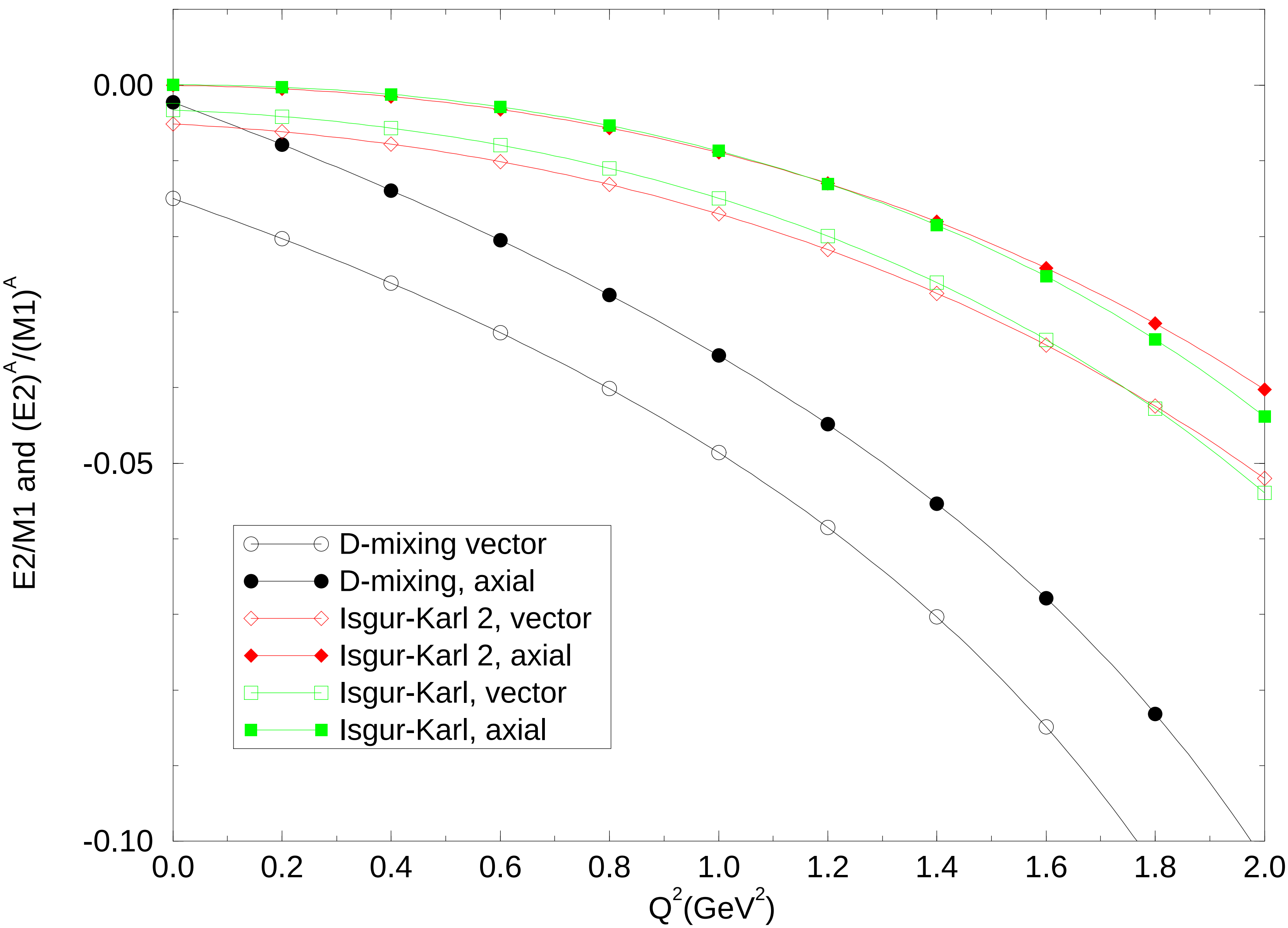


0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0.0
0.5
1.0
1.5
2.0
C_4^A
$Q^2(GeV^2)$
D-Mixing
Isgur-Karl
Isgur-Karl 2
SU(6)
Kitagaki-Adler


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
C_5^A
0.0
0.5
1.0
1.5
2.0
$Q^2(GeV^2)$
Kitagaki-Adler
D-Mixing
Isgur-Karl
Isgur-Karl 2



0.05
0.00
-0.05
-0.10
-0.15
0.0
0.5
1.0
1.5
2.0
C_6^A
$Q^2(GeV^2)$
D-Mixing
Isgur-Karl
Isgur-Karl 2



0.00
-0.05
-0.10
E2/M1 and (E2)^A/(M1)^A
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
Q^2(GeV^2)
D-mixing vector
D-mixing, axial
Isgur-Karl 2, vector
Isgur-Karl 2, axial
Isgur-Karl, vector
Isgur-Karl, axial